

under the "usual" operations of analysis. The student is familiar with continuous functions and with the difficulties due to the fact that limits of convergent sequences of such functions are not necessarily continuous. It is thus easy to make him understand the importance and value of the closure property. Furthermore, it seems useful to avoid the possibility of the student's acquiring the misleading idea that the measurability concept is based on the measure concept, and also to point out that these two concepts are related only through the background of completely additive classes. However, the main reason for the suggested approach is didactic. This approach requires a few concepts only, hence it is easier to digest for the student who is not overburdened with new concepts. Also, since convergence theorems are basic in the modern theory of integration and in its use, it seems preferable to get to them as fast as possible and not to have to cover the first 220 pages. Construction of measures and study of metric spaces, which are far more abstract and involved, would come later.

Stripped of the concepts and details unnecessary to the approach outlined above, the part of the book relative to measurability, integration, and convergence theorems would make an excellent text for a one-semester course, possibly an undergraduate one. The foregoing remarks reflect mainly the fact that this well-written and well-rounded book may be used successfully in various ways.

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*Contributions to the theory of games.* Vol. 2. Ed. by H. W. Kuhn and A. W. Tucker. (Annals of Mathematics Studies, no. 28.) Princeton University Press, 1953. 396 pp. \$4.00.

This collection of various papers on the theory of games is the second such volume edited by Professors Kuhn and Tucker to form an Annals of Mathematics Study. The merit of this form of publication seems to the reviewer to be considerable, because the great extent and development of mathematics make a volume dedicated to a specialized topic an efficient way to reach specialists. A good many of the twenty-one papers in the volume are good and indicate that the subject has not yet lost its vitality or momentum.

The first section of the volume contains five papers on finite zero-sum two-person (z-s t-p) games. Paper no. 1, by v. Neumann, discusses the optimal assignment problem, i.e., the assignment of  $n$  persons to  $n$  jobs so as to maximize the value of the assignment. He shows that the problem is equivalent to solving a z-s t-p game which is simpler computationally than the original assignment problem. Paper no. 2 by Gillies, Mayberry, and v. Neumann, discusses two

variants of poker. In paper no. 3 Motzkin, Raiffa, Thompson, and Thrall give a method for computing optimal solutions. In paper no. 4 Dresher and Karlin construct an algorithm for the solutions of games where the mixed strategies of the players constitute compact convex sets (in Euclidean space); the solutions are fixed points of appropriate mappings. In paper no. 5 Arrow, Barankin, and Blackwell discuss "admissible" points of a convex set.

The second section of the book is devoted to  $z$ - $s$   $t$ - $p$  games where the players dispose of infinitely many pure strategies, e.g., where the pure strategies of a player are points in an interval. Shiffman in paper no. 6 and Karlin in paper no. 7 study games of timing, i.e., games on the unit square, say, where the payoff function is increasing in one variable and decreasing in the other. The name comes from the fact that the pure strategies may be interpreted as times at which the players act, and it is profitable for each player to delay his own action as long as possible, provided he acts in advance of his opponent. In paper no. 8 Karlin studies a class of games where the payoff function has a positive  $n$ th partial derivative with respect to one of the variables; this is a natural extension of the notion of a convex game. Glicksberg and Gross in paper no. 9 exhibit some games with rational payoff functions whose pathologic behavior disappoints the hope that certain properties of finite games could be extended to games with rational payoff functions. In the problem of statistical decision as formulated by Wald (and in certain other  $z$ - $s$   $t$ - $p$  games), several writers have shown under various hypotheses the equivalence of mixing strategies before and after the chance variable (say) is observed. In paper no. 10 Blackwell studies a special case where any randomized strategy is equivalent to a mixture in fixed proportions of countably many pure strategies.

The third section of the volume is devoted to games in extensive form. In paper no. 11 Kuhn discusses a new formulation of the extensive form of a general  $n$ -person game which covers a larger class of games than the formulation of v. Neumann. He uses a geometric model, the successive presentation of alternatives in the game appears as the branching of a topological tree, and the patterns of information appear as partitions of the vertices of the tree. Several general results are proved for certain games in this form. (In his excellent book *Introduction to the theory of games*, McKinsey made use of Kuhn's formulation.) Dalkey (paper no. 12) adopts Kuhn's formulation and studies "variations in the structure of a game in extensive form which leave the major strategic properties of the game invariant irrespective of the payoff function." His paper

extends to general games results of Krental, McKinsey, and Quine (Duke Math. J. vol. 18 (1951) pp. 885–900). In paper no. 13 Gale and Stewart study z-s t-p games with perfect information (chess is such a game). It was proved by v. Neumann (e.g., v. Neumann and Morgenstern, *Theory of games and economic behavior*, 2d. ed., Princeton University Press, 1947, p. 112; see also E. Zermelo, *Ueber eine Anwendung der Mengenlehre auf die Theorie des Schachspiels*, Proceedings of the Fifth International Congress of Mathematicians, Cambridge, 1912, vol. II, p. 501) that every such game whose number of moves is finite has a solution in pure strategies. Gale and Stewart consider games where the number of moves is infinite, and among their results is the construction of a game with perfect information which has no solution in pure strategies. In paper no. 14, G. L. Thompson studies signaling strategies and applies his results in paper no. 15 to a model of the game of bridge. A signaling strategy for a player is “a pure strategy for that player restricted to that subset of his information sets which prevent him from having perfect recall.” In paper no. 16, J. W. Milnor analyzes a situation which occurs in certain games, where “one can measure the ‘incentive’ to move at any particular configuration by imagining the possibility of passing instead.”

The fourth and final section is devoted to  $n$ -person games. In paper no. 17 L. S. Shapley proposes to evaluate the equities of the players of an arbitrary  $n$ -person game. Whether such a game has a solution in the sense of v. Neumann and Morgenstern is an unsolved problem. In papers no. 18, no. 19, and no. 20, R. Bott, D. B. Gillies, and L. S. Shapley, respectively, introduce interesting classes of games for which they obtain solutions. In paper no. 21 H. Raiffa proposes “arbitration conventions” for choosing an imputation from the solution set of v. Neumann and Morgenstern.

Each section of the volume is preceded by an excellent editorial introduction which summarizes the various papers and indicates lines of further research. The volume itself is indispensable for students of the subject.

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#### NEW JOURNALS

*Mathematica Scandinavica*. Vol. 1, no. 1. Copenhagen, 1953. 192+12 pp. 40 Danish crowns per volume of two numbers; 20 crowns to members of the sponsoring societies and members of societies (including the American Mathematical Society) having reciprocity agreements with them.