BOOK REVIEWS


This is the first book in the English language on the topic. In fact,
while many aspects of stability theory have been summarized, not­
ably by Russian authors, no mathematically satisfactory treatise on
the subject has appeared since the classical work of Liapounoff. It is
the reviewer’s feeling, therefore, that any new book on stability must,
at least partially, be judged on how it summarizes and organizes the
highlights of the work done in this field since Liapounoff.

Bellman’s book is organized in seven chapters. The first and third
chapters are introductions to the theory of linear and nonlinear dif­
ferential equations, respectively. Matrix and vector notation is used
throughout these two well-written chapters. Chapter two deals with
questions of stability, boundedness, and asymptotic behavior of
linear differential equations, many of them originally investigated by
Bellman himself. Chapter four is the heart of the book. Three dif­
ferent proofs are given for the stability theorem for the case where
the linearized system has constant coefficients. The proofs are clear.
A clever counterexample is provided to show that one cannot hope
to extend the theorem without modification to the case where the
linearized system does not have constant coefficients. In chapter
five the asymptotic behavior of certain first order nonlinear equa­
tions is considered. Chapter six deals with linear second order equa­
tions. After an introduction to the more elementary properties of
linear equations the author proves certain theorems about the
boundedness in norm of solutions using various norms. Chapter six
closes with a study of the oscillatory and asymptotic behavior of the
solutions. Finally, chapter seven discusses the linear second order
equation,

\[
\frac{d}{dt} \left( \rho \frac{du}{dt} \right) \pm t^\sigma u^\mu = 0.
\]

It is shown that certain conclusions with regard to the nature of
solutions can be drawn from a knowledge of \( \rho, \sigma \) and \( \mu \).

The book as a whole is well-written and quite readable. There are
many exercises to supplement the text. However, it seems unfortu­
nate that the author excluded certain aspects of stability theory which, in
a book dedicated to this field, seem more important than chapters
five and seven. There is hardly anything in the book, for instance, on the stability of periodic solutions, or in the sixth chapter on the second order linear equation with periodic coefficients. There is comparatively little reference to work done in the last ten years either in this country or abroad. Aside from these omissions, however, Bellman's book is a pleasant and interesting contribution to the theory of differential equations.

F. Haas


Although self-regulating devices have been in operation since the days of the governor on Watt's steam engine, it is only in recent years that the subject of automatic control has assumed a central position in the engineering and industrial world.

From the mathematical side, the control problem leads to systems of nonlinear differential equations in the following way. If we assume that the state of the physical system is specified at time \( t \) by the vector \( x(t) \), the study of small displacements from equilibrium gives rise, in a system without control, to a linear vector-matrix equation \( \dot{x} = Ax \). If we now consider a system with control, where the control is manifested by a forcing term and the magnitude of the control is dependent upon the state of the system, the resulting equation for \( x \) has the form \( \dot{x} = Ax + f(x) \), and is, in general, nonlinear.

The term "continuous control" will be used to describe situations in which \( f(x) \) is a continuous function of \( x \). In many cases, it was found that continuous control was far too expensive to use. In place then of control devices which gave rise to forcing terms of continuous type, it was far cheaper to design control devices yielding forcing terms whose components are step-functions of \( x \). The simplest version of this type of control system is one with a simple on-and-off control mechanism. This type of control is called "discontinuous automatic control."

A simple example of the mathematical equations which result is the following second order equation, \( \ddot{u} + a \dot{u} + bu = c \) \( \text{sgn} \ (\dot{u} + ku) \), where \( u \) is now a scalar function. This equation has the form \( \ddot{u} + a \dot{u} + bu = c \), over the region of phase space described by \( \dot{u} + ku > 0 \), and the form \( \ddot{u} + a \dot{u} + bu = -c \), over the region of phase space described by \( \dot{u} + ku < 0 \). If \( \dot{u} + ku = 0 \), the forcing term is taken to be zero.

We observe then the interesting fact that while the equation itself is nonlinear, over the regions \( \dot{u} + ku \geq 0 \), \( u \) may be determined as a solution of a linear equation, albeit a different linear equation over different regions.