great importance in the study of transonic flow. The applications to this theory are given in several sections.

Two very useful features are the ample references to both quite recent and classical papers and the eighty or so representative problems gathered at the ends of the various sections.

The book is particularly to be recommended to anyone intending to work in the mathematical theory of hydrodynamics or aerodynamics.

RICHARD BELLMAN

Infinite abelian groups. By I. Kaplansky. (University of Michigan Publications in Mathematics, no. 2.) Ann Arbor, University of Michigan Press, 1954. 5+91 pp. $2.00.

The theory of finite and infinite abelian groups is comparatively rich in structure theorems and these form the central theme of Kaplansky's excellent monograph. A satisfactory characterization is available, and presented here, for the groups in the following classes of abelian groups: finite groups [Frobenius-Stickelberger], torsion groups with bounded order, finitely generated groups, countable torsion groups [Ulm], groups with division, countably generated torsionfree modules over complete discrete valuation rings [Kaplansky]. Direct sums of cyclic groups are naturally prominent in this discussion, since the groups in the classes mentioned are either direct sums of cyclic groups or direct sums of cyclic groups and groups with division or else contain direct sums of cyclic groups as essential building blocks. Thus one finds here the theorems of Prüfer and Kulikoff, characterizing certain direct sums of cyclic groups, and their application proving that every subgroup of a direct sum of cyclic groups is itself a direct sum of cyclic groups.

Various structures may be derived from an abelian group. The ring of endomorphisms is known to reflect particularly faithfully the properties of the original group; and the author proves for a large class of groups that they are completely determined by their endomorphism rings—this is one of many instances where results in this work go beyond what had been known before. Similarly the theory of characteristic subgroups has been developed considerably further than had been done by the author's predecessors.

The first eleven chapters lead up to Ulm's theorem which is proved with admirable simplicity. This is no mean task, considering the difficulty of Ulm's original proof and of Zippin's simplified version thereof. Operators make their first appearance in the 12th chapter, quite rightly in our opinion, since in a large part of the theory of abelian groups the operators do not contribute any more to our discussion than the—purely ring theoretical—question: which properties of the integers have been really needed for our arguments? Quite naturally the answer to this question does not throw too much light on the theory of abelian groups. Still the operators have their uses: firstly a special selection of the ring of operators may produce a class of abelian groups that is amenable to treatment—a trivial example is provided by the modules over a field; and secondly they open the way to interesting applications and the author has not missed his opportunities here.

This book is written with admirable simplicity and lucidity; and it is a pleasure to be led by the author through this field, formerly so inaccessible, now so easy of access. Here it should be noted that the interesting and deep results presented in this book had been scattered through the mathematical periodicals of the world if they had been published at all. But let the reader not be misled by the apparent slimness of this monograph: There are a hundred theorems given without proof [under the misleading name of exercise] and if their proofs had been supplied [instead of the "hints" given in difficult situations] the size of the book would have been doubled at the least.

A substantial bibliography with a useful "guide to the literature"—that might be considered as a rather concentrated report on the state of the theory of abelian groups—deserves our particular attention.

We have explained why we believe that the student of the theory of abelian groups will be grateful to the author for this work. But there are various reasons why this book will be a great help to those of us who attempt to educate mathematicians and to introduce young people to the present-day ways of mathematical thinking. For instance, one likes to impress on a student's mind the central importance of structure theorems. But there do not exist many of them; and without examples of structure theorems it is impossible to explain their significance and in particular what constitutes a satisfactory structure theorem. Now these theorems form, as we mentioned before, the central theme of the book under review. Next the transfinite tools, so helpful in various branches of mathematics, are used here extensively so that the reader will learn how to avail himself of these methods. As a matter of fact this was one of the author's aims in writing this book; and in this as in his other aims he has succeeded brilliantly.

REINHOLD BAER

Handbook of elliptic integrals for engineers and physicists. By P. F. Byrd and M. D. Friedman. (Die Grundlehren der mathematischen