The reviewer pondered this question at some length and offers the following conjecture. There are several closely related geometries in the Cartesian plane, the confusion of which can be as troublesome as the confusion of different concepts of variable, *especially in the mathematical representation of physical quantities*, or "dimensional analysis" in the physicist's sense. The principal geometry of the Cartesian plane for the purposes of measurement is that of the Cartesian product of the affine line and affine line $A_1 \times A_1$ in which independent transformations, $x' = ax+b$, $y' = cy+d$, $a \neq 0$, $c \neq 0$, are admitted. Physical laws must have statements invariant in this geometry. The subject of dimensional analysis lives in this geometry. However, if in apparently innocuous indulgence, the more familiar Euclidean plane geometry E_2 is adopted instead, the subject of dimensional analysis vanishes, leaving behind only ghosts whose goings on are to be recorded by fiat. The author is not explicit about his choice of geometry but the reviewer's conjecture is that his point of view about "physical variables" is that of a man who is thinking in E_2.

In the opinion of the reviewer a consistent operator approach to the representation of physical quantities in the elementary sense is still needed to prepare the way for this point of view which forces itself upon us of necessity in more advanced situations. The author of this book has laid the groundwork for it by introducing the identity function in lieu of the scalar variable, so that his linear operators $\mathcal{D}$ and $\mathcal{D}^{-1}$ work properly. The natural extension of this same idea should do much to simplify dimensional analysis.

Another interesting aspect[1] is that the variable used by Menger is much closer to the idea of a random variable than the conventional one, in fact, *is* a random variable with uniform distribution.

WILLIAM L. DUREN, JR.

Elements of number theory. By I. M. Vinogradov. Trans. from the 5th rev. ed. by S. Kravetz. New York, Dover, 1954. 8+227 pp. $1.75 paper; $3.00 cloth.

This elementary book was originally published in 1936 with a little under 100 pages and subsequent editions appeared every few years until the sixth in 1952 of almost double the size. This work, as most other Russian books, has not been easy to obtain in the past and its present appearance is gratefully noted. It is a very welcome addition to the books on number theory.

As has been observed by other reviewers, the main body of the

[1] Pointed out by R. N. Bradt.

text, containing a mere 77 pages, is largely of a standard nature. The remainder of the work, aside from tables listing indices, primes, and primitive roots, is devoted to the statement and solution of problems. These are almost 100 in number with many that have several parts; their statement occupies about 55 pages and their solution 84 pages. In addition, there are about 40 numerical exercises, their answers being given at the end of the book.

The problems, therefore, constitute the chief novelty of the work. As might be expected, these are very heavily slanted in the direction of the author's outstanding researches on exponential sums and their applications. Although the methods used in the solutions are elementary, many of the problems are too difficult for beginning students. On the whole, the book may be considered as an excellent introduction to the author's monograph *The method of trigonometrical sums in the theory of numbers* now also translated into English. As can be seen from the discussion below, the number of topics is limited and even such a classical topic as Diophantine equations only barely makes its appearance in the form of a few problems.

Chapter I, *Divisibility theory*, treats such material as the greatest common divisor, Euclid's algorithm and its relation to simple continued fractions, fundamental properties of these fractions, and the unique factorization theorem. Among the problems, one finds the Diophantine equations $x^2+y^2=z^2$, $x^4+y^4=z^4$, and the basic properties of Farey series.

Chapter II, *Important number-theoretic functions*, deals with the factorization of $n!$, with the functions $[x]$, $x-[x]$, $\sigma_s(n)$, $\mu(n)$, $\phi(n)$, and with the general properties of multiplicative functions. The problems are concerned with various identities connecting number-theoretic functions, problems on lattice points, the almost indispensable Euler-Maclaurin sum formula (here ascribed to Sonin), estimates and asymptotic formulas for various sums, and the Möbius inversion formula in sum and product form.

Chapter III, *Congruences*, deals with basic properties, complete and reduced residue systems, and the theorems of Fermat and Euler. A number of the problems are concerned with sums of the form $\sum_x \{f(x)-[f(x)]\}$ where x ranges over an interval, while others treat the related question of the number of lattice points in certain regions. Other problems are concerned with exponential sums.

Chapter IV, *Congruences in one unknown*, sets forth the continued fraction solution of the linear congruence, the usual theorems on congruences with composite and prime power moduli, and Wilson's theorem. The problems deal with the number of solutions of congruences, the Kloosterman sums and with various aspects of congruences.

Chapter V, *Congruences of the second degree*, contains the standard material on the Legendre and Jacobi symbols as well as the solution of the congruence $x^2 \equiv a \pmod{m}$. Among the unusual problems, one finds many on the sum of various types of Legendre symbols, the Gaussian sum $\sum_{x=1}^{m} e^{2\pi i a x^2/m}$ as well as other exponential sums.

Chapter VI, *Primitive roots and indices*, is concerned with the determination of all moduli having primitive roots and with the corresponding theory of indices. For such moduli, the congruence $x^n \equiv a \pmod{m}$ is treated. The modulus 2^α, which has no primitive root if $\alpha > 2$, is considered and the essentially unique representation $a \equiv (-1)^\gamma 5^{\gamma_0} \pmod{2^\alpha}$ is obtained when $2 \nmid a$. A large number of the problems deal with characters, character sums, and exponential sums including the Kloosterman sums.

The translation reads smoothly except for a number of exceptions, and the typography is an improvement over the original even though the right-hand margins of the pages have not been rectified. Although the errors mentioned in an insert to the sixth edition have been corrected, a considerable number have been introduced and some others have not been caught. For example, there are a number of places where the numeral 1 has been used in place of the letter l (pp. 35, 125, 133). In other places, lower case letters have been used in place of upper case letters (p. 87) or vice versa (p. 34). In at least three places (pp. 128, 128, 214), the sign $=$ is used in place of the correct sign $\leqq$. There are other errors as well. Also, unfortunately, the chapter and paragraph titles have been omitted from the tops of pages. It is to be hoped that the publisher will correct these deficiencies in a subsequent printing.

LOWELL SCHOENFELD

Enzyklopädie der mathematischen Wissenschaften. Vol. I, *Algebra und Zahlentheorie*. Part 1: A. *Grundlagen*; B. *Algebra*. 2d ed. Vol. I_1, no. 3, part I: I_1, 6, *Lineare Algebra*; I_1, 7, *Normalformen von Matrizen*. By G. Pickert. Leipzig, Teubner, 1953. 72 pp. 7.50 DM.

The new edition of the *Enzyklopädie*, which was long delayed by the war, seems now to be getting well under way, at least for the first parts, covering foundations, algebra and number theory. While retaining essentially as their goal the same one as the first edition, namely to give as complete as possible a description of the mathematics of their day, the editors have wisely dropped all the historical