

ferential equations and membrane and conducting sheet analogies.

The book begins with chapters on mechanical and electrical computing elements which are followed by a description of machines for simultaneous linear algebraic equations and a chapter on nonlinear equation solvers, harmonic analyzers, and the conduction sheet analogy for the complex plane. The mechanical and electrical differential analyzers are each treated in a chapter. The concluding chapters develop the analogies between dynamical and electrical systems, the analogies on which the finite difference solutions of partial differential equations are based, and the above mentioned analogies for the membrane and conducting sheets.

As an engineering text book this appears to be excellent. The derivations are specific to the application and at the mathematical level associated with college elective courses on differential equations. This book would also be a good introduction to analogies for the mathematician interested in the myriad mathematical problems associated with this field.

FRANCIS J. MURRAY

*The kinematics of vorticity.* By C. Truesdell. Bloomington, Indiana University Press, 1954. 20+232 pp. \$6.00.

This is a work packed with theorems of a general type concerning the vorticity of a fluid. The author uses freely the classical theory of vectors in three dimensions, the theory of dyadics and, to some extent, the tensor calculus. After various geometrical preliminaries, and the definitions of velocity, acceleration, expansion, deformation, etc., of a fluid motion, the vorticity is defined and its various interpretations are given. The vorticity field and the notions of vortex lines and tubes are next discussed, with their bearing on circulation. The measure of vorticity is given a chapter to itself and this is followed by one on vorticity averages. Bernoullian theorems are then investigated, by which are meant formulae for the squared speed of the fluid and the scalar potential of the flow. The two final chapters deal with the convection and diffusion of vorticity and with circulation-preserving motions.

I have attempted to estimate the number of theorems contained in the book and have concluded that there is probably an average of two per page, making a total of perhaps 400. Though they are expressed in the terminology of hydrodynamics, they are essentially theorems in pure mathematics, expressing relations between vectors and their integrals over volumes and surfaces. That the vectors are called velocity, acceleration, vorticity, etc., is not an essential feature

of the treatment. One is brought up therefore against the question: What is the central problem of applied mathematics? To one school of thought, it is the establishment of a body of theorems and methods in pure mathematics that might conceivably be useful in dealing with physical situations. A hydrodynamist is an expert in vector analysis, who may never have watched water flowing; a relativist is an expert in differential geometry who may never have heard of the perihelion of Mercury. According to this school, Truesdell's book is an excellent monograph on hydrodynamics, compact, succinct and packed with information. But there is another school which holds that the central problem consists of sizing up the physical situation, of estimating what are the essential variables, and what the relations between them may be. In some twenty years' experience of teaching applied mathematics—not to speak of the attempts to carry out my own researches!—I have noticed that it is the translation of a physical situation into mathematical terms which is found difficult by students. The subsequent pure mathematics are a relatively minor obstacle. From this point of view Truesdell's book suffers from the defect that the reader is given no clue as to whether—if at all—any particular result or theorem is applicable to a physical situation nor, if it is, what the nature of the situation may be. The name of Horace Lamb often occurs in the text, as is to be expected, and it is interesting to compare an example of Truesdell's exposition with Lamb's. The former, in §72, considers "Lamb surfaces" which are simultaneously vortex and stream surfaces; that such surfaces exist in any circulation-preserving motion with steady vorticity is proved as a theorem in pure mathematics and there the matter is left. The reader is given no clue as to whether such motions are ever encountered in nature and, if they are, under what circumstances. In contrast, when Lamb proves the same theorem (H. Lamb, *Hydrodynamics*, 5th ed., Cambridge University Press, 1924, §165), he immediately produces two illustrations of the kinds of physical situation wherein it might be useful. To Lamb, pure mathematics is a tool, to Truesdell it would seem to be an end in itself. In defence of the latter's method it must be said that illustrations greatly extend the length of a book and that fourteen pages of references should surely be a sufficient guide to possible physical applications.

A curious feature of Truesdell's treatment of vorticity is his extreme insistence on the importance of three spatial dimensions (see, for example, the remarks on pp. 3, 59, 77). It is true that if vorticity is to be expressed as a *vector*, three dimensions are necessary; but the spinning or rotational characteristics of a fluid can hardly be de-

pendent on the particular mathematical form used to express them. Indeed, if a 4-dimensional treatment is employed the vorticity is no longer a vector but is described by three of the components of a second-rank antisymmetrical tensor. Evidently Truesdell means that his particular formalism, in terms of 3-vectors, is valid only in three dimensions which is no doubt the case; but over-emphasis on this point of view leads to remarks such as that on p. 77, relative to Lagrange's acceleration formula. Whilst it is true that the particular formula (38.2) holds only in 3 dimensions, it is also true that a corresponding formula, involving the vorticity tensor, holds in 4 dimensions. Thus it does not appear to be correct to state without qualifications that "for the existence of Lagrange's formula it is requisite that the number of dimensions be three."

But when all has been said, one important fact emerges: this book is a valuable compendium of results that every expert in hydrodynamics, gas-dynamics or dynamical meteorology will want to keep by his side and refer to frequently.

G. C. McVITTIE

*Introduction to integral geometry.* By L. A. Santaló. Paris, Hermann, 1953. 127 pp. 1500 fr.

Integral geometry is the name given by Blaschke to a branch of geometry which originated with problems on geometrical probabilities and deals with relations between measures of geometrical figures. One should perhaps consider as its father the English geometer W. F. Crofton, while Poincaré left an important mark by introducing the kinematic measure (and by writing a book on geometrical probabilities).<sup>1</sup> In 1934 Blaschke and his coworkers started a series of papers on the subject. The author of this book made some of the most beautiful contributions to it in that period and, in the twenty years which followed, has consistently added new results to it. It is therefore gratifying that a book by the author now exists in the literature. The book is elementary in nature. Its first two parts presuppose only some knowledge of calculus and the last part some projective geometry and a little more maturity.

Part I, on the metric integral geometry of the plane, studies the densities of points, lines, pairs of points, etc. and culminates with Poincaré's kinematic density and Blaschke's kinematic formula. One of the most interesting applications is the author's proof of the

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<sup>1</sup> As a matter of historic interest mention should be made of a set of lecture notes by G. Herglotz, which introduced many of the early workers into the subject.