BOOK REVIEWS

Some basic problems of the mathematical theory of elasticity. By N. I. Muskhelishvili. 3d, rev. and augmented ed., trans. from the Russian by J. R. M. Radok. Groningen, Noordhoff, 1953. 31+704 pp. 38.00 florins.

The three-dimensionality of space is one of those things the applied mathematician has to put up with. It is not merely that three is greater than two and so offers more complications; the trouble seems to lie rather in the oddness of three which hinders an effective algebraic approach. Thus after one is tired of playing with the equations of three-dimensional elasticity in the best available notation, the tensorial, and having little to show in the way of problems actually solved, one turns with relief to plane problems in which the complex variable may be used; here we have not only notational compactness; we have access to a rich store of mathematical knowledge accumulated for other purposes.

The application to elasticity of the theory of the functions of complex variables has been largely the work of Russian mathematicians, and this book gives us an authoritative account, written by one who has himself contributed much. As the title of the book indicates, it is not a treatise on the whole body of elasticity but a discussion of certain problems, which are plane problems or problems which can be reduced to such.

The book had its origin in lectures delivered in 1931–32 in the Seismological Institute of the Academy of Sciences of the U. S. S. R. (but it contains nothing about elastic vibrations) and the Physico-Mathematical Institute of Mathematics and Mechanics at the University of Leningrad. The first edition appeared in 1933, the second in 1935, and the third (of which the book under review is a translation) in 1948, the general design of exposition remaining unchanged (the author says) throughout the several editions. The preface by Academician A. N. Krylov to the first edition is included; this ends as follows:

"There only remains to express the wish that in future editions, which without doubt will be required, the author illustrate the general deductions and formulae by numerical examples, by diagrams and by indications as to the number of ordinates or subdivisions required for approximate integration in order to ensure accuracy within, say, ½%. He will thereby render a great service to engineers and make his excellent book more accessible to those people who will apply its deductions to the solution of the purely practical problems of the building industry."

Happily (in the opinion of the reviewer) the author does not seem to have taken this advice too seriously. The book is a mathematician's book, using consistently a powerful mathematical method (complex variable) and, although there are many practical applications, one might say that the engineer is expected to bring his own cup to the well instead of having it piped to him.

The historical notes show, at times, a rather petulant insistence on the importance of the work of Russians. One does not question this; the elegant and systematic application of complex variable theory to elasticity seems to be their achievement. But it is perhaps a little too much to describe Love's famous Treatise as "in many respects obsolete," particularly as the author makes no fewer than nineteen references to it.

However this is a minor matter. It does not mar what is indeed an excellent book, well printed, with formulae generously spaced and easy to read, and (what is more important) written in an uncramped style which carries the reader along smoothly without leaving in his mind a trail of half-formulated queries. It is a clear book. Paper has not been parsimoniously saved, and one judges that the translator has done his work well.

There are seven Parts, three Appendixes, authors' index, references, and subject index. The last is of course too brief; they always are.

Part I covers the usual three-dimensional theory of stress, strain, and Hooke's Law. It is done in the traditional clumsy notation, but this is no fault, because this three-dimensional theory is only presented to make the book self-contained. In Part II we pass to the general formulae of plane theory (plane strain and generalized plane stress, essentially one mathematically) with the biharmonic stress function and its complex representation, fundamental boundary value problems and their reduction to problems in complex function theory. Multivalued displacements (dislocations) are discussed and thermal stresses. Also conformal transformations with examples, well illustrated by diagrams.

Part III gives solutions by power series (circle and circular hole, concentrated forces, circular ring, etc.) and solutions by conformal mapping. Parts IV and V deal with Cauchy integrals and their application to boundary value problems, in particular for regions mapped on a circle by rational functions (many particular cases considered); also semi-infinite regions.

Part VI treats regions with cuts in them under the general heading of the problem of linear relationship, alias the Riemann problem or (as the author would prefer) the Hilbert problem, the problem being to find a sectionally holomorphic function $F(z)$ with a line of discontinuity L, the boundary values of which from the left and from the right satisfy the condition

$$F^{+}(t) = G(t)F^{-}(t) + f(t) \text{ on } L$$

(except at the ends), where $G(t)$ and $f(t)$ are functions given on L and $G(t) \neq 0$ everywhere on L.

Part VII becomes a little more three-dimensional, dealing with extension, torsion and bending of homogeneous and compound bars.

This is a book to be recommended in the highest terms to every serious student of the mathematical theory of elasticity. And to engineers also, for though they may find some of the work too purely mathematical for their taste, they will be rewarded by solutions of definite problems completely worked out. The reviewer was tickled by the enlivenment imparted to the solution of the torsion problem for a circular cylinder reinforced by an eccentric bar; one part of the solution is due to the cylinder itself and the other part to the "indignation" aroused by the presence of the reinforcement!

J. L. SYNGE

Vorlesungen über Differential- und Integralrechnung. Vol. III. *Integralrechnung auf dem Gebiete mehrerer Variablen*. By A. Ostrowski. Basel, Birkhäuser, 1954. 475 pp., 36 figs. 78 Swiss fr.; paper bound 73.85 Swiss fr.

This is the third and concluding volume of Professor Ostrowski's comprehensive text on calculus. For the reviews of volumes I and II, both by the present writer, the reader is referred to Bull. Amer. Math. Soc. vol. 52 (1946) pp. 798–799 and vol. 58 (1952) pp. 513–515. In this third volume there are seven chapters, all of them dealing to some extent with integration. There is an index for volumes II and III, a glossary of symbols for these two volumes, and a short list of corrections for volumes I and II.

Chapter I deals with the technique of integration. The topics are: complex numbers, partial fractions and integration of rational functions, integration of algebraic and transcendental functions, and the transcendence of e. The discussion of partial fractions is complete, with proofs.

Chapter II, on the definition of multiple integrals, is actually one third devoted to "the general case" of integrals of functions of one variable. The author first discusses sets of zero Jordan content, here-