sented as the sums of two \( k \)th powers, and the reflection of electromagnetic waves on a parabolic cylinder, show the wide range of applications of these functions.

A table of Laplace transform pairs involving confluent hypergeometric functions, the list of references mentioned above, and an index conclude this excellent volume.

The printing is excellent, and very few misprints were noted by this reviewer. Tricomi's book is the first volume in a new series of mathematical monographs sponsored by the Italian Consiglio Nazionale della Ricerche. It is a promising beginning, and in wishing the new venture every success, one can hardly do better than express the hope that future volumes of the series will be as useful and as readable as Tricomi's book.

A. ERDÉLYI


The subtitle and almost every page of this little book hold forth promise to mathematicians on the alert for radical new applications of mathematics, but I fear that they will be rather disappointed. The specific subject matter seems dry and relatively unimportant for mathematicians and biologists alike; the method chosen by the author seems ineffective and tedious; there are scarcely any interesting arguments or deductions presented or alluded to; and the final results are meager.

The general problem of classifying a vast set of objects has many aspects of intellectual and perhaps of mathematical interest, especially when the objects are organisms, with their philogenetic connections. This book is confined to discovering a description of the formal, set theoretic, structure of the biological taxonomic systems in actual use, completely abstracted from how these systems depend on the structure and philogeny of organisms. For example, it is within the scope of the book to say that a species is a set of organisms and a genus a set theoretic union of species but not to say when two organisms should be assigned to a common species or genus.

As the author says, biologists have over the centuries evolved a remarkably accurate and effective language for describing the forms of organisms, but they have as yet no special language for talking about taxonomic systems, as opposed to talking taxonomy. The author seeks to supply the missing language with the aid of symbolic logic. I believe he would have done far better to look to a powerful,
accurate, flexible, living language that, like biological nomenclature, has been evolving over the centuries, namely the everyday language of mathematical publication. For example, the author presents with some fanfare the following definition attributed to Woodger:

\[(3.1) \quad (HR = \text{the set in which any relation } s \text{ is a member if and only if } (s \in \text{One-Many}) \text{ and } (\tilde{s}''\text{UN} = \tilde{s}'\text{B}'\tilde{s}')).\]

Translation: An HR (hierarchical relation) is the relation of immediate-predecessor-to in a partial ordering in which there is a unique initial element and in which every other element has a unique immediate predecessor.

Again, consider the definition of taxonomic classificatory system, TCS, from which many theorems are said to flow.

\[(5.6) \quad (TCS = \text{the set in which any relation } s \text{ is a member if and only if } (s \in \text{HR}) \text{ and there is some } u \text{ such that } (u \subseteq \text{FIELD}'z) \text{ and } (s = \text{IC}(u)) \text{ and } (u \subseteq \text{G}) \text{ and, for all } x, \text{ all } y, \text{ and all } w, \text{ if } ((x; y) \subseteq \text{IC}(u)) \text{ and } ((x; w) \subseteq \text{IC}(u)) \text{ and } (y \neq w) \text{ then } ((y \cap w) = \text{EM}).\]

This seems to mean the relation of immediate inclusion in a class of sets of organisms, \(A, B, C, \cdot \cdot \cdot\), such that this relation is hierarchical and such that if \(A \cap B \neq 0\) then \(A \subseteq B\) or \(B \subseteq A\). The author says in a footnote that since formulating (5.6) he has discovered a better definition, which he couches in language much more like ordinary mathematical language.

Two justifications of the author's very formal terminology are implicit in his statement that (3.1) is "a notation more favorable for calculating and for methodological discourse in general." Now there is almost no calculation or formal deduction in the book and little is called for, because the subject matter is logically so simple. As for the second justification, the notation does not seem to me nearly as favorable as ordinary mathematical language. The author himself almost always provides some translation into that language, and without it the book would be practically unintelligible.

I think it was a minor methodological step in the wrong direction for the author to adopt the theory of types, which mathematicians have long preferred to live without. Incidentally, he apparently forgets that he has adopted it in formulating several of his definitions, e.g., (1.15), (2.8), (2.17). He does encounter one difficulty in which the theory of types might have been put to some use. Even if, for example, a genus has only one species, taxonomists do not like to say that the genus and the species are the same. One solution would be
to admit that, though in one sense a genus and a species are both sets of organisms, there is another sense in which a genus is a set of species. Personally, I think the difficulty is even better resolved by reference to intentional and extensional definition.

I have said that the results are meager. Indeed (5.6) and its exploration seem to be the high point of the book.

There are few errors, of which the most interesting is this. To illustrate the concept of incompatible relations, it is asserted that “if $x$ is father of $y$ then $x$ is not cousin [of any order] of $y$, and vice versa.” Actually, it is incestuous (nephew-aunt mating) for a man to beget a first cousin, but not impossible; moreover, I conjecture that most fathers and their sons are distant cousins.

To sum up, the book attacks a problem of minor interest with some success, success that would have been greater had the author been able to interest mathematicians in his project while it was under way.

Leonard J. Savage


The theory of “best approximation,” i.e. of minimizing the maximum of the absolute difference between a given function and its approximating function, has been studied extensively ever since it was initiated by Chebyshev, but practical approximations have in the past usually been based on other, analytically more manageable, methods. It appears from this book (implicitly, since no motivation is given explicitly) that Chebyshev approximation is just what is required in work with digital computing machines. Apparently one gives the machine a simple polynomial or rational approximation, which it can easily evaluate, in preference to an exact transcendental function. It also appears that almost none of the elaborate theory of Chebyshev approximation is of any practical use, and that the construction of useful approximations is still much more of an art than a science. Indeed, it is only rarely that the “best” approximation can be found explicitly, and the computer must usually be satisfied with some other approximation which is still uniformly close enough for his needs. The first part of the book consists of the reproduction of a strip film, with running commentary, describing the art of concocting approximations as practiced by an acknowledged expert. The second part presents 76 numerical approximations to 23 varied functions, each with a carefully drawn graph of the error. The book is evidently addressed to computer technicians rather than to mathematicians.