RESEARCH PROBLEMS


Given \( N \) coins, \( k \) of which are defective, either lighter or heavier than the other \( N-k \) coins which are assumed to be of equal weight, and a balance, determine the weighing procedures which minimize the number of weighings required to separate the defective coins from the ordinary coins. Consider the following two cases
   a. The \( k \) defective coins are all of the same weight, heavier or lighter than the regular coins.
   b. The \( k \) defective coins are all of different weight.

Also determine the weighing procedures which minimize the expected time required to determine the defectives.

This is a particular case of the general "sorting" problem where an individual element of a set may be characterized by a number of properties and we have a number of testing devices for determining these properties. (Received May 31, 1955.)

22. Richard Bellman: \textit{Analysis}.

The derivative of the gamma function satisfies the recurrence relation

\[
\Gamma'(x + 2) = (2x + 1)\Gamma'(x + 1) + x^2\Gamma'(x),
\]

for \( x > 0 \). Can one derive from this equation a convergent continued fraction expansion for \( \Gamma'(x)/\Gamma'(x+1) \), or a related expression, which can be used either
   a. To obtain a rapid method for computing \( \Gamma'(1) \), the negative of Euler's constant, or
   b. To obtain some results concerning the arithmetic character of Euler's constant? (Received May 31, 1955.)

23. Richard Bellman: \textit{Number theory}.

There are a number of numerical techniques available for determining the maximum over the \( x_i \) of the linear form, \( L(x) = \sum a_i x_i \), subject to the linear constraints

\[
\sum b_i x_i \leq c_i, \quad i = 1, 2, \ldots, M,
\]

whenever it exists. Can one obtain a usable algorithm for the cases where we impose additional constraints of the form
   a. \( x_i = 0 \) or \( 1 \), for \( i = 1, 2, \ldots, N \), or
   b. \( x_i \) is zero or a positive integer? (Received May 31, 1955.)

24. Sherman Stein: \textit{Number theory}.

Let \( a \) be a positive rational fraction with odd denominator and \( u_n = (2n+1), n = 1, 2, \ldots \). Let \( b_1 \) be the smallest of the \( u_i \) satisfying \( a - (u_i)^{-1} \geq 0 \). Having defined \( b_1, b_2, \ldots, b_n \), define \( b_{n+1} \) as the smallest \( u_i, u_i > b_n \), with \( a - (b_i)^{-1} - \cdots - (b_{n+1})^{-1} \geq 0 \).

Is the sequence \( b_1, \ldots, b_n, \ldots \) finite for each \( a \)? (Received May 23, 1955.)

25. Sherman Stein: \textit{Geometry}.

Let \( J \subset R_2 \) be a rectifiable Jordan curve, with the property that for each rotation \( R \), there is a translation \( T \), depending on \( R \), such that \( (TRJ) \cap J \) has a nonzero length. Must \( J \) contain the arc of a circle? (Received May 23, 1955.)