contained a minor error, and that he had never seen 50 pages of mathematical work without a serious mistake. The author now informs the reviewer that a mimeographed list of errata for this book can be obtained by writing to him.

J. KOREVAAR

Probability theory. Foundations. Random sequences. By M. Loève. Toronto, Van Nostrand, 1955. 15+515 pp. $12.00.

The mathematical theory of probability has been well established in its modern form for about twenty years, but Loève's is the first (non-elementary) textbook on the subject. That is to say, there has been no textbook which, like a textbook covering any other advanced mathematical subject, gives the fundamental definitions, basic theorems, and enough further development to lead the reader into the really advanced literature. Feller's *Probability theory and its applications* (1950) is superlative as far as it goes (only through discrete probabilities), but the promised further volumes are still keeping company with the other unborn descendants of first volumes, an illustrious group. The reviewer's *Stochastic processes* (1953) has been the closest approach to a probability textbook, but, as its title indicates, this book was neither written as nor intended to serve as a general textbook, and its choice of topics and emphasis were dictated by its title. Authors have been willing to write specialized probability books, of which there have been many, besides the two just mentioned, by Bartlett, Blanc-Lapierre and Fortet, Fortet, Gnedenko and Kolmogorov, Ito, Lévy, but the drudgery involved in writing a systematic and complete text has not been attractive. Thus, even if Loève's book were not as successful as it is, he would still deserve the thanks and respect of the mathematical community for writing it.

The mathematical theory of probability is now a branch of measure theory, with certain specializations and emphasis derived from the applications and the historical background. As the historical conditioning loses its significance for newer generations of mathematicians, the place of probability theory in measure theory becomes more and more difficult to describe. One slightly frivolous description, which, however, is about as accurate a description as can be given, is that probability is the one branch of measure theory, and in fact the one mathematical discipline, in which measurable functions as such are considered in detail, and their integrals evaluated. (The fact that the integration of smooth functions on intervals can be considered as that of measurable functions is of course discounted here.) In fact, in most applications of measure theory, say to the study of L_2 spaces, or to potential theory, the abstract measure theory serves to make the final results elegant—to make certain spaces complete, and so on, but any integrals actually evaluated are integrals that could have been evaluated by Euler. The point is that probability theory is more than just an application of measure theory. It is an intrinsic part of the theory, which uses the general ideas so intensively that the probabilist must have at his disposal concepts which students are unlikely to study in a first course in measure theory. The writer of a probability text must therefore either devote part of the book to measure and integration, covering more than is usually covered in a first course, or assume as known the content of such a course and develop the subject from there, or, under the same assumption, simply list a long collection of assorted results developed in different notations in different places. The second and third choices would be very difficult to carry through in a satisfactory way, and Loève has wisely chosen the first. He has thus written a book which, assuming the usual prerequisites to a first course in measure theory, as well as a considerable amount of mathematical maturity, conducts the reader through a remarkably thorough treatment of abstract measure and integration theory, and an extraordinary coverage of the basic concepts and results of probability theory. (The statement, in an otherwise humorless book, that "the prerequisite is calculus" should be taken in the spirit in which it was presumably intended.) The book has as subtitle *Foundations. Random sequences* and there is correspondingly no discussion of continuous parameter stochastic processes (c.p. "random functions" in Loève's terminology) except for the material on second order random functions.

Loève could not include so much material in his book without some sacrifice, and in fact he has, to some extent, sacrificed motivation, specialization and application of general results, and readability. Only the individual readers can decide whether the coverage is worth this sacrifice, but the extremely terse style is frequently not easy to follow. Loève's definitions are exact and receive no commentary even when they contain easily overlooked details which may be critically important. His theorems are tightly linked together, so that every statement can be and is justified simply by a reference to a previous theorem. He frequently does not make the specializations that would illuminate the subject. The classical discrete problems and applications, treated so beautifully in Feller's book, are absent, and in fact there are essentially no applications, except to other parts of mathematics. In other words, and this is or is not a criticism according to the preferences of the reader, the book is purely a mathematical text. (There is, however, a reference to a second volume, and the remarks on omissions are subject to correction if this volume breaks a tradition by appearing.)

The book begins with an introductory section, intended to provide intuitive meaning to the concepts and problems of probability theory, although this section is rather abstractly written. It is an independent section of the book, and in fact is so independent of the remainder that when, in a later chapter, Markov chains with enumerably many states are discussed, and a result is proved which is more special than one proved in the introductory section, no reference is made to the earlier result, or even to the earlier treatment! The introductory section contains a discussion of the basic probability definitions in the discrete case, and the work is carried through the central limit theorem and strong law of large numbers for Bernoulli trials, and the Kolmogorov treatment of Markov chains with enumerably many states and stationary transition probabilities. The first two chapters, constituting *Part One*, cover measure and integration. Chapters III and IV, constituting *Part Two*, cover the standard fundamental concepts of probability theory: random variables, distributions, characteristic functions, convergence. Chapters V and VI, constituting *Part Three*, concentrate on problems connected with independent random variables: convergence of series, the laws of large numbers, the central limit problem, including a detailed discussion of infinitely divisible and stable distributions. Chapters VII–X, constituting *Part Four*, involve dependent random variables. Chapter VII takes up conditional probabilities and expectations, giving among other things a careful discussion of when conditional probabilities can be assumed to define completely additive set functions. There is a very elegant treatment of the Markov dependence property. Chapter VIII shows how the theorems of Chapters V and VI can be extended to dependent random variables, and takes up martingales, proving the essential convergence theorems. Chapter IX covers ergodic theorems, including sufficient material on ergodic theorems for Banach space operators to obtain the Doeblin theorems on convergence of iterated conditional probabilities, following Yosida and Kakutani. Chapter X studies second order properties, including considerable material on second order random functions. The stationary case is studied as a special case.

A bibliography is appended to the introduction, to each of the four parts, and to each chapter. These separate bibliographies represent the sources of the author's material, but are by no means slavishly followed. The arrangement of bibliographies and the references make it difficult in some cases to find out exactly which work is the source for a given result, and therefore where further material may be found.

The above short outline does not suggest the remarkable amount of material in this book. The author's compact style enables him to include an extraordinary mass of ideas and theorems in a systematic and coherent fashion. At each stage he usually succeeds in achieving the utmost in generality, both in the large and in minor details—for example in allowing infinite-valued expectations wherever possible. This book will be a standard reference text for years, and students will find it indispensable, although difficult.

J. L. Doob

General topology. By J. L. Kelley. New York, Van Nostrand, 1955. 14+298 pp. $8.75.

The appearance of a comprehensive treatise in English on present day set-theoretic topology is an important event. The rapid progress in set-theoretic topology during the past 20 years, and the ever increasing applications of this discipline to analysis, make the appearance of the volume under review particularly appropriate at the present time. Professor Kelley has set himself the task of producing a book useful for both students and specialists, and he has succeeded to a remarkable extent in reaching both of these somewhat inconsistent goals. Like many other texts of this genre, the present volume could be understood, at least in theory, by any intelligent person who can read English. All of the machinery is supplied; but a knowledge of the real numbers and elementary abstract algebra as set forth for example in Birkhoff-MacLane (*A survey of modern algebra*, rev. ed., New York, Macmillan, 1953) and a thorough knowledge of elementary analysis are certainly minimal prerequisites for appreciating this book.

The author's style is spirited, to say the least. The atmosphere of an informal and humorous lecture pervades the book, especially in the first part. An essential difference between oral and written communication is well illustrated here, for some sallies that would clearly enliven a lecture are less felicitous in print. (For example, a very special case of Fubini's theorem is designated without further comment as Fubinito (p. 78), and a *topologist* is defined as a man who doesn't know the difference between a doughnut and a coffee cup (p. 88).) An occasional solecism was noted (e.g., p. 135, line 7), and one might wish that the author adhered to the "which" and "that" precepts of Fowler. However, these are but trifling criticisms of a