original proof. In Chapter 4, the subject of study is the class of power series with unit radius of convergence, and various justifications of the general remark that most of these have the unit circle for a cut; for example [Boerner], the set of $\beta=(\beta_0, \beta_1, \cdots)$ with $|\beta_j| \leqq \pi$ for which $\sum a_n \exp (i\beta_n)z^n$ is continuable has measure zero, while [Pólya-Hausdorff] the class of non-continuable power series is open and dense in the space of power series.

Another general remark of somewhat similar nature is that whenever $\{a_n\}$ is a sufficiently "nice" sequence, $\sum a_n z^n$ is either rational, or has the circle of convergence for a cut. This is illustrated by the material of Chapter 6 which revolves around the classical theorems of Eisenstein, Szegö, Pólya, and Carlson. As an instance of the method of associated functions, the author digresses in §6.3 to include a sketch of some known results dealing with integral valued entire functions. If $A(z)$ is an entire function of exponential type such that $A(n) \in D$ for $n=0, 1, \cdots$ where D is a domain of algebraic integers, then one may consider the associated function $g(z) = \sum_0^\infty A(n)z^n$. If A is of sufficiently slow growth, $g(z)$ is regular in a set of mapping radius greater than 1; if D is either the rational integers, or a quadratic complex domain, then the Pólya-Carlson theorem may be applied to show that $g(z)$ is a rational function, from which the form of $A(z)$ may be obtained. Finally, Chapter 5 deals with certain consequences of the Hadamard multiplication theorem, and Chapter 7 discusses the connection between a power series $\sum a_n z^n$ and the related series $\sum \phi(a_n)z^n$ where $\phi(z)$ is a preassigned analytic function.

The author has not entered upon the general coefficient problems for schlicht functions and for bounded functions, nor has he discussed analytic continuation of power series by means of summability. Within his chosen framework, he has produced a remarkably interesting and coherent summary of recent work and literature.

R. C. BUCK

Theory of differential equations. By E. A. Coddington and N. Levinson. New York, McGraw-Hill, 1955. 14+429 pp. $8.50.

It has become fashionable of late, in various mathematical centers, to present the fundamental tools of analysis, real and complex variable theory, in an increasingly abstract manner to those most defenseless, namely fledgling graduate students. In the process, motivation for the introduction of new concepts has been on the whole by-passed as an atrophied relic of those early pioneer days when mathematicians were forced to consort with astronomers and physicists, and indeed, in some cases, were indistinguishable from them. It may be true, howsoever improbable, that mutation or inbreeding has produced a new type of mathematical mind that can absorb winged words before pedestrian ones. It is doubtful, however, whether this species can perpetuate itself. Most likely, mathematical education will continue along the same simple logical principles that have guided the great scientists of the past, from the simple to the complex, from the concrete to the abstract. If these premises be accepted, a very good case can be made for choosing the subject of differential equations, with its ramifications, as a basic course of study for first-year graduate students.

To begin with, existence and uniqueness theorems furnish an excellent testing ground for introducing new concepts and techniques and for training the skeptical mind. Three fundamental methods of analysis, successive approximations, finite differences, and fixed-point techniques, all enter in very natural fashion. The last method may serve, if one wishes, as an introduction to the general study of differential and integral operators and functionals. Alternately, in connection with finite difference and successive approximations, one enters the new domain of the study of numerical methods in relation to the optimal use of high-speed computers.

Turning to the study of specific classes of equations, matrix theory enters as the elegant approach to the treatment of linear equations. Those with constant or linear coefficients afford an excellent showcase for the Laplace transform. The consideration of basic physical problems leads to Sturm-Liouville problems. By way of the basic principle of superposition, we encounter Fourier series and Fourier integrals, these last occasionally requiring Stieltjes integrals. Alternate approaches lead to the theory of integral equations, this time of Fredholm type, and to Rayleigh-Ritz principles, a domain of great richness.

The concept of generalized solution and summability methods enter as soon as Fourier expansions are used to solve initial value and boundary problems. Furthermore, complex variable methods are an essential tool, following Cauchy, in discussing the expansion problem for general Sturm-Liouville problems. Expansion of the solution of a linear equation with polynomial coefficients as a power series introduces the concept of asymptotic series, and expansion in terms of a parameter appearing in the equation may be treated by means of the same ideas.

Finally, we enter the nonlinear domain, where the principle of superposition is a wistful memory. Immediately, we encounter the stability problem, a paramount concept in the study of functional equations. The study of periodic solutions of differential equations, of particular significance in the nonlinear case, leads, following the footsteps of Poincaré, to the study of families of curves and surfaces, and so into the field of topology. Systems with more than two degrees of freedom introduce almost-periodicity as the suitable substitute for the more restrictive periodicity.

With differential equations as a springboard, practically the whole sea of analysis lies before one.

If the above program sounds interesting, the book by Coddington and Levinson on differential equations will serve as an admirable text as far as both range and virtuosity are concerned. In any case, the book stands as an excellent and comprehensive survey of much of the modern theory of differential equations.

Let us now summarize the contents of the volume. The first two chapters are devoted to the question of the existence and uniqueness of solutions, and the dependence of the solution upon parameters and initial values. The next four chapters contain a study of linear systems, with constant, periodic and rational coefficients. A number of results due to the authors in connection with the asymptotic behavior of solutions is contained in these chapters. In particular, there is an interesting application of Phragmén-Lindelöf theorems to the study of asymptotic series in the complex plane. Those interested in parts of the theory of linear equations with periodic coefficients not covered in the present text may refer to V. M. Starzinskii, *A survey of works on the conditions of stability of the trivial solution of a system of linear differential equations with periodic coefficients*, American Mathematical Society Translations Project, series 2, vol. 1.

The following five chapters contain a discussion of Sturm-Liouville problems, for both second order and general nth order equations, including a chapter devoted to non-self-adjoint problems, an area of great current interest. The method used goes back to Cauchy, depending upon Green's function and the method of residues. The details, however, are far from trivial, and much of the content of these chapters was developed by the authors. The next chapter is devoted to an exposition of the stability theory of Poincaré and Liapounoff, as developed and perfected by Perron. There is, however, no reference to the great volume of work done by the Russian school since Liapounoff, contained in the papers of Malkin, Persidiski, and others. In two succeeding chapters, there is a detailed study of the perturbation theory of systems with periodic solutions. The penultimate chapter presents the Poincaré-Bendixson theory of two-dimensional orbits, and the last chapter treats the difficult topic of periodic solutions on a torus.

An important feature of the book is the inclusion of approximately one hundred and seventy-seven problems of varying degrees of difficulty, with hints as to the solution. This greatly increases the scope of the book. Finally, let us note that the book is printed in the attractive easy-on-the-eyes style which we have grown to expect from McGraw-Hill.

RICHARD BELLMAN

Elemente der Funktionalanalysis. By L. A. Ljusternik and W. I. Sobolew. (Mathematische Lehrbücher und Monographien, vol. 8.) Berlin, Akademie-Verlag, 1955. 10+253 pp. 25.00 DM.

This book is a translation of *Elementy funkcional'nogo analiza* [Gostehizdat, Moscow-Leningrad, 1951], which was reviewed in Mathematical Reviews in Vol. 14 (1953) p. 54. It is an introduction to the theory of normed linear spaces and operations on them, and contains few surprises. It is more complete and more sophisticated than *Elementy teorii funkciĭ i funkcional'nogo analiza* by Kolmogorov and Fomin [Izdatel'stvo Mosk. Universiteta, 1954], and is totally different from *Leçons d'analyse fonctionnelle* by Riesz and Sz.-Nagy [Akadémiai Kiadó, Budapest, 1952]. There are many points of contact with Banach's classical monograph *Théorie des opérations linéaires* [Monografje Matematyczne, Warszawa, 1932]. So far as the reviewer knows, there is no single treatise in English covering the same material as the book under review.

The reader is tacitly expected to know the elementary theory of functions of a real variable: continuity; differentiation; functions of finite variation; Lebesgue integration on $[0, 1]$. For students with this background, the book is highly recommended as an introduction to functional analysis.

Chapter I deals with metric spaces. The only nonstandard topic here is Banach's theorem on contracting mappings: a mapping A of a space X (with metric ρ) into itself such that $\rho(Ax, Ay) \leqq \alpha\rho(x, y)$ for all $x, y \in X$ and some α, $0 < \alpha < 1$, admits exactly one fixed point. Several applications of this theorem are given. Chapters II and III deal with linear spaces, linear operators, and linear functionals; this part bears a strong family resemblance to Banach's book. Several interesting and not universally known facts are given: for example, if E is a Banach space with conjugate space $\overline{E}$, and if E is not reflexive, then

$$E, \overline{E}, \overline{\overline{E}}, \overline{\overline{\overline{E}}}, \ldots$$