and the last chapter treats the difficult topic of periodic solutions on a torus.

An important feature of the book is the inclusion of approximately one hundred and seventy-seven problems of varying degrees of difficulty, with hints as to the solution. This greatly increases the scope of the book. Finally, let us note that the book is printed in the attractive easy-on-the-eyes style which we have grown to expect from McGraw-Hill.

RICHARD BELLMAN

Elemente der Funktionalanalysis. By L. A. Ljusternik and W. I. Sobolew. (Mathematische Lehrbücher und Monographien, vol. 8.) Berlin, Akademie-Verlag, 1955. 10+253 pp. 25.00 DM.

This book is a translation of *Elementy funkcional'nogo analiza* [Gostehizdat, Moscow-Leningrad, 1951], which was reviewed in Mathematical Reviews in Vol. 14 (1953) p. 54. It is an introduction to the theory of normed linear spaces and operations on them, and contains few surprises. It is more complete and more sophisticated than *Elementy teorii funkciĭ i funkcional'nogo analiza* by Kolmogorov and Fomin [Izdatel'stvo Mosk. Universiteta, 1954], and is totally different from *Leçons d'analyse fonctionnelle* by Riesz and Sz.-Nagy [Akadémiai Kiadó, Budapest, 1952]. There are many points of contact with Banach's classical monograph *Théorie des opérations linéaires* [Monografje Matematyczne, Warszawa, 1932]. So far as the reviewer knows, there is no single treatise in English covering the same material as the book under review.

The reader is tacitly expected to know the elementary theory of functions of a real variable: continuity; differentiation; functions of finite variation; Lebesgue integration on $[0, 1]$. For students with this background, the book is highly recommended as an introduction to functional analysis.

Chapter I deals with metric spaces. The only nonstandard topic here is Banach's theorem on contracting mappings: a mapping A of a space X (with metric ρ) into itself such that $\rho(Ax, Ay) \leqq \alpha\rho(x, y)$ for all $x, y \in X$ and some α, $0<\alpha<1$, admits exactly one fixed point. Several applications of this theorem are given. Chapters II and III deal with linear spaces, linear operators, and linear functionals; this part bears a strong family resemblance to Banach's book. Several interesting and not universally known facts are given: for example, if E is a Banach space with conjugate space $\overline{E}$, and if E is not reflexive, then

$$E, \overline{E}, \overline{\overline{E}}, \overline{\overline{\overline{E}}}, \ldots$$

are all different (Plessner). Banach's theorem that $C([0, 1])$ is universal for separable Banach spaces is stated and proved. Chapter IV, dealing with completely continuous operators, is standard and also good. Chapter V contains a standard discussion of the spectral theorem for self-adjoint bounded operators on a separable Hilbert space. Chapter VI, the last in the book, is credited mainly to Ljusternik. It deals with nonlinear functional analysis. The principal topics are: derivatives and integrals for functions on $[0, 1]$ with values in a Banach space; Fréchet's differential; implicit function theorems for Banach-space valued functions; extreme values and their calculation.

Limitations of the book in subject matter, perhaps justified in an introductory textbook, are the following. The L_p-spaces dealt with are all on $[0, 1]$. The only space of continuous functions considered is $C([0, 1])$. Compactness for nonmetric spaces is ignored: in discussing the unit ball in $\overline{E}$, for example, the authors show only that it is weakly sequentially compact if E is separable—an assertion much weaker than the theorem of Alaoglu-Bourbaki. Countability arguments are used wherever possible, and the necessary appeal to transfinite induction in proving the Hahn-Banach theorem is slurred over.

The book is fairly discursive, and should be easy reading. It is beautifully printed. The translation is on the whole good, although it is misleading in a few places. A couple of errors in the Russian original have been quietly corrected. The book is marred, however, by a ridiculous exaggeration of the rôle played by Russian, and in particular by Soviet, mathematicians in the development of functional analysis. The rule adopted seems to be: if some Russian had anything to do with it, mention him and no one else; if not, mention no one if you can help it.

EDWIN HEWITT

BRIEF MENTION

Einführung in die Verbandstheorie. By H. Hermes. Berlin, Springer, 1955. 8+160 pp. 22.80 DM.

The quickest way to describe the book under review is to say that if there were a Bourbaki treatment of lattice theory, it would be pretty much like that of Hermes. (It is unlikely that there ever will be a Bourbaki treatment of the subject; cf. Bull. Amer. Math. Soc. vol. 59 (1953) p. 483.) The book is an introduction to the elements of lattice theory; the exposition is handled with painstaking care and thoroughness. Once the author decides to discuss a subject, he discusses it systematically; his treatment, for instance, of the various