

are all different (Plessner). Banach's theorem that $C([0, 1])$ is universal for separable Banach spaces is stated and proved. Chapter IV, dealing with completely continuous operators, is standard and also good. Chapter V contains a standard discussion of the spectral theorem for self-adjoint bounded operators on a separable Hilbert space. Chapter VI, the last in the book, is credited mainly to Ljusternik. It deals with nonlinear functional analysis. The principal topics are: derivatives and integrals for functions on $[0, 1]$ with values in a Banach space; Fréchet's differential; implicit function theorems for Banach-space valued functions; extreme values and their calculation.

Limitations of the book in subject matter, perhaps justified in an introductory textbook, are the following. The L_p -spaces dealt with are all on $[0, 1]$. The only space of continuous functions considered is $C([0, 1])$. Compactness for nonmetric spaces is ignored: in discussing the unit ball in \bar{E} , for example, the authors show only that it is weakly sequentially compact if E is separable—an assertion much weaker than the theorem of Alaoglu-Bourbaki. Countability arguments are used wherever possible, and the necessary appeal to transfinite induction in proving the Hahn-Banach theorem is slurred over.

The book is fairly discursive, and should be easy reading. It is beautifully printed. The translation is on the whole good, although it is misleading in a few places. A couple of errors in the Russian original have been quietly corrected. The book is marred, however, by a ridiculous exaggeration of the rôle played by Russian, and in particular by Soviet, mathematicians in the development of functional analysis. The rule adopted seems to be: if some Russian had anything to do with it, mention him and no one else; if not, mention no one if you can help it.

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BRIEF MENTION

Einführung in die Verbandstheorie. By H. Hermes. Berlin, Springer, 1955. 8+160 pp. 22.80 DM.

The quickest way to describe the book under review is to say that if there were a Bourbaki treatment of lattice theory, it would be pretty much like that of Hermes. (It is unlikely that there ever will be a Bourbaki treatment of the subject; cf. Bull. Amer. Math. Soc. vol. 59 (1953) p. 483.) The book is an introduction to the elements of lattice theory; the exposition is handled with painstaking care and thoroughness. Once the author decides to discuss a subject, he discusses it systematically; his treatment, for instance, of the various

kinds of homomorphisms (join, meet, lattice, order) is remarkable for its completeness and clarity. There are a few exercises, but they are not very numerous or exciting. The book is divided into five chapters, as follows: (I) Foundations; (II) The simplest classes of lattices; (III) Modular lattices; (IV) Distributive and Boolean lattices; (V) Miscellany. The fifth chapter discusses Zorn's lemma, congruence relations, and some connections between Boolean algebra and logic. There is an appendix that treats in great generality the universal algebraic concepts (e.g., homomorphism) of which special cases are treated in the body of the book. There is nothing new here for the expert, but a beginning student might find the treatment a pleasant illustration of some of the easier techniques of algebra.

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RESEARCH PROBLEMS

7. H. E. Salzer: *Numerical analysis*.

It is known that I: an n th degree polynomial is determined by $(n+1)$ values, considering its value and the *first consecutive* r derivatives at a point as $(r+1)$ values. The writer found for $(2n-1)$ th degree formulas of the type (1) $f_{[n/2]+1} = \sum_{i=-[n/2]}^{[n/2]} A_i f_i + h^2 \sum_{i=-[n/2]}^{[n/2]} B_i f_i'$ the existence of solutions A_i, B_i for $n=2$ and 4, but not for $n=1$ (trivial), 3, or 5, suggesting the following:

(a) In general is a polynomial of $(2n-1)$ th degree determined by its value and its *second* derivative at n equally-spaced points if and only if n is even?

(b) What generalizations of I are there, involving $(r+1)$ *non-consecutive* derivatives (value considered as 0th derivative)? Thus the writer found $(2n)$ th degree formulas of the type (2) $f_{[n/2]+1} = \sum_{i=-[n/2]}^{[n/2]} A_i f_i + h \sum_{i=-[n/2]}^{[n/2]+1} B_i f_i'$ for $n=1(1)5$, indicating that f can be determined from n values of f and f' at n points and f' at an $(n+1)$ th point. (Received December 5, 1955.)

8. H. E. Salzer: *Numerical analysis*.

The integration formula (1) $\int_{-1}^1 f(x)/(1-x^2)^{1/2} dx = (\pi/n) [f(\cos \pi/2n) + f(\cos 3\pi/2n) + \dots + f(\cos (2n-1)\pi/2n)]$ is exact for $f(x)$ any $(2n-1)$ th degree polynomial, so that the weight function $1/(1-x^2)^{1/2}$ gives a Gaussian-type quadrature formula for a finite interval, which happens to be equally-weighted. Can one find weight functions $w(x)$ and $W(x)$ yielding Gaussian-type quadrature formulas of the form (2) $\int_0^\infty w(x)f(x)dx = (a/n) \sum_{i=1}^n f(x_i)$ and (3) $\int_{-\infty}^\infty W(x)f(x)dx = (b/n) \sum_{i=1}^n f(x_i)$, combining the advantage of orthogonality or algebraic degree of precision equal to $2n-1$ with equal weights? Unlike (1), do solutions to (2) and (3) exist only when $w(x)$ and $W(x)$ depend upon n ? (Received December 5, 1955.)