

depends only on the cohomology class to which α belongs. The map which sends each cohomology class into the corresponding μ -asymptotic cycle is a homomorphism. In particular, if we insist on a single-valued Hamiltonian function, so that $\alpha = dH$, the μ asymptotic cycle is zero. (Received January 12, 1956.)

395. Stephen Smale: *A Vietoris mapping theorem for homotopy.*

Let X and Y be compact metric spaces and $f: X \rightarrow Y$ be onto. Vietoris proved (Math. Ann. (1927)) that if for all $r \leq n-1$ and all $y \in Y$, $H_r(f^{-1}(y)) = 0$ (Vietoris homology mod 2) then $f_*: H_r(X) \rightarrow H_r(Y)$ is an isomorphism onto for $r \leq n-1$ and onto for $r = n$. Begle (Ann. of Math. (1950)) gave generalizations to nonmetric spaces and more general coefficient groups. Examples show that this theorem does not hold directly for homotopy. However, suppose X and Y are LC^n and that for each $y \in Y$, (1) $f^{-1}(y)$ is LC^{n-1} and (2) $\pi_r(f^{-1}(y)) = 0$ for $0 \leq r \leq n-1$; then it is proved that $f_\#: \pi_k(X) \rightarrow \pi_k(Y)$ is an isomorphism onto for $k \leq n-1$ and onto for $k = n$. Actually the hypotheses can be weakened in some respects. For example, if instead of assuming $\pi_{n-1}(f^{-1}(y)) = 0$, one only has that $i_\#: \pi_{n-1}(f^{-1}(y)) \rightarrow \pi_{n-1}(X)$, the homomorphism induced by inclusion, is zero, then still $f_\#: \pi_{n-1}(X) \rightarrow \pi_{n-1}(Y)$ is an isomorphism onto. (Received December 14, 1955.)

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RESEARCH PROBLEM

9. Richard Bellman. *Minimization problem.*

We are given a region R and a random point P within the region. Determine the paths which

- (a) Minimize the expected time to reach the boundary, or
- (b) Minimize the maximum time required to reach the boundary.

Consider, in particular, the cases

- (a) R is the region between two parallel lines at a known distance d apart.
- (b) R is the semi-infinite plane and we are given the distance d from the point P to the bounding line. (Received November 18, 1955.)