Notes, which were widely circulated but which have been unavailable for quite some time.

Chapter 6, which deals with three-dimensional problems, is also new. The basic approach involves the expression of the components of displacement in terms of four arbitrary harmonic functions. Treated here are cases of concentrated loading, the problem of Boussinesq, the equilibrium of the sphere, thermoelastic problems, vibration problems and others.

G. E. HAY

La géométrie des groupes classiques. By Jean Dieudonné. Berlin, Springer, 1955. 7+115 pp. 19.60 DM.

This book gives an excellent survey of recent work on classical groups, simplifying and unifying the results of many authors. No attempt is made to cover all of the voluminous literature on classical groups; the author deals with only that portion of the subject which can be handled by the methods of linear algebra. By thus restricting his scope, he is able to include proofs of most of the results described, thereby making the book more self-contained than most Ergebnisse tracts.

While the book is written on an advanced level, it presupposes only some familiarity with linear algebra. However, a reader with a minimum background will have to work hard to master this book, which cannot be skimmed lightly. By use of a highly-condensed method of presentation, and omission of many routine details of proofs, the author has succeeded in packing a large amount of information into relatively few pages. The average reader will want to keep pencil and paper handy, in order to work through most of the proofs. There were several places where this reviewer would have been grateful for a few extra lines of exposition.

Chapter I (Collineations and correlations, pp. 1–35). By a *collineation* of an n-dimensional vector space E over a skew-field K is meant a one-to-one semi-linear map of E onto itself. The group $\Gamma L_n(K)$ of all such collineations contains the group $GL_n(K)$ of linear one-to-one maps of E onto itself. The "projective" groups are defined as groups modulo their subgroups of homothetic maps ($x \rightarrow xa$, $a \in K$). The beginning sections take up the concepts of dilatations and transvections (these are collineations leaving a hyperplane pointwise fixed), involutions and semi-involutions (these are collineations u for which $u^2(x) = x$ or xc ($c \in K$), respectively), and their centralizers in $P\Gamma L_n(K)$, the group of projective collineations.

By a *correlation* is meant a one-to-one semi-linear map ϕ of E onto its dual E^*, relative to an anti-automorphism J of the skew-field K. A non-degenerate form f is introduced on $E\times E$ by means of $f(x, y) = \langle\phi(x), y\rangle$ (inner product), and it is assumed that f is *reflexive* (that is, $f(y, x)=0$ if and only if $f(x, y)=0$). The fundamental notions of hermitian and skew-hermitian forms, orthogonal complements, isotropic spaces, symplectic and orthogonal bases, and equivalence of forms follow next.

Call a collineation $u\in\Gamma L_n(K)$, with automorphism σ, a *unitary semi-similitude* relative to a form f if $f(u(x), u(y)) = r_u(f(x, y))^\sigma$, where $r_u\in K$. The group $\Gamma U_n(K, f)$ of all such u contains the subgroup of unitary transformations $U_n(K, f)$ consisting of those u for which $r_u=1$ and σ is the identity. For hermitian forms, it is shown that it suffices to consider only *trace* forms (forms f for which $f(x, x)=a+a^J$, $a=a(x)\in K$). Chevalley's proof of Witt's theorem is given for such forms.

There is next a discussion of the dilatations (called *quasi-symmetries* in this case) and transvections in $\Gamma U_n(K, f)$, and the problem of centralizers of semi-involutions in $P\Gamma U_n(K, f)$ is considered. (These sections are rather difficult reading.) The chapter concludes with a section on projective permutability of correlations, and one on orthogonal groups over fields of characteristic 2.

Chapter II (Structure of the classical groups, pp. 36–72). (The following results are established except for certain special cases, which are described in the book.) Let $SL_n(K)$ be the subgroup of $GL_n(K)$ generated by transvections. It is shown that $SL_n(K)$ is the commutator subgroup of $GL_n(K)$, and the structure of their quotient is determined. A proof of the simplicity of $PSL_n(K)$ is given. The analogous problems are then considered for the unitary and orthogonal groups. The author shows that $U_n(K, f)$ is generated by quasi-symmetries, and the structures of $U_n(K, f)$, $T_n(K, f)$ (subgroup of U_n generated by unitary transvections, assuming that f has positive index), and U_n/T_n are determined. The reader is reminded very briefly of what Clifford algebras are, and then these are used to derive results on the orthogonal group $O_n(K, f)$ and its commutator subgroup $\Omega_n(K, f)$. A new proof of the simplicity of $P\Omega_n(K, f)$ is given.

Chapter III (Geometric characterization of the classical groups, pp. 72–85). Let $G_r(E)$ be the Grassman space consisting of all $(r+1)$-dimensional subspaces of E. Let E' be a vector space over K', with $[E':K']=[E:K]$; then any one-to-one semi-linear map u of E onto E' induces a map of $G_r(E)$ onto $G_r(E')$. The problem of characterizing such induced maps is solved here for $r=0$ by the fundamental theorem of projective geometry, and for $r>0$ by use of the concept of "adjacent" elements of the Grassman space. The same type of problem is also considered for $u\in U_n(K,f)$. The chapter concludes with a brief discussion of other characterizations of the classical groups.

Chapter IV (Automorphisms and isomorphisms of the classical groups, pp. 85–108). The automorphisms of $GL_n(K)$, $SL_n(K)$, $Sp_{2m}(K)$ (symplectic group), $U_n(K,f)$, and the corresponding projective groups are determined. The methods used are quite elegant and non-computational. Many details are omitted, but adequate references are given to cover these gaps. In each case, the structure of the proof is clearly described. There is also a discussion of the possible isomorphisms between the various classical groups.

Following these chapters is a table of notations used in the book, a table comparing these notations with those of Dickson (*Linear Groups*) and van der Waerden (*Gruppen von Linearen Transformationen*), an index of principal theorems and definitions, and a useful bibliography which brings up to date that given in the above-mentioned book of van der Waerden. The book is well-printed, although the style of using solid paragraphs of exposition with little displayed material is a bit trying.

The author has asked that the following corrections be pointed out. P. 11, line -9: $(f(y\xi, x))^J$. P. 13, lines -15 to -13, replace by the sentences: Un sous-espace V de E qui n'est pas isotrope est caractérisé par le propriété que le sous-espace orthogonal V^0 est supplémentaire de V; mais un tel sous-espace peut contenir des droites isotropes. On dit que V est *anisotrope* s'il ne contient pas de droites isotropes; un tel espace est évidemment non isotrope. P. 23, line -11: $U_p(K,f_1)$. P. 26, line 3: $vu=uv\cdot a$. P. 33, line 8: $2p+d+1\leqq i \leqq n \cdots \sum_{i=1}^{n}$. P. 36, middle of part (a): $B_{ij}(1)B_{ji}(-1)B_{ij}(1)$. P. 36, last sentence in part (a), and p. 37, lines 21–23: these are correct only when K is commutative. P. 37, lines -8 to -5, replace by the sentence: si $A=(a_{ij})$ et si $a_{i1}\neq 0$, en retranchant des lignes de A d'indice $\neq i$ des multiples à gauche de la i-ème ligne, on obtient une matrice B dont la première colonne n'a que a_{i1} comme term $\neq 0$; on pose alors $\det(A)=\phi((-1)^{i+1}a_{i1})$ det (B_{i1}), en désignant par B_{i1} la matrice obtenue en supprimant dans B la première colonne et la i-ème ligne. P. 72, line -14: $\cdots u$ de E sur $E' \cdots$. P. 72, line -7: $\cdots$ par $\phi \cdots$. P. 107, line -16: read "$m=n=1$" for "$m=n=2$." P. 111, insert before line -4: ABE, M.: [1] Projective transformation groups over non-commutative fields, Sijo-Sugaku-

Danwakai, 240 (1942). P. 114, line 13: read "J. Reine Angew. Math., 196" for "Math. Zeitschrift."

To summarize: in the reviewer's opinion, this is an important and well-written book which should help to stimulate research on the classical groups. The book not only gives a thorough exposition of the present state of the subject, but is also an excellent introduction to the modern techniques basic to further work in this field.

IRVING REINER

Relativity: The special theory. By J. L. Synge. New York, Interscience, 1956. 7+415+20 pp. $10.50.

In the preface of this book Synge states, "The basic idea of this book is to present the essentials of relativity from the Minkowskian point of view, that is, in terms of the geometry of space-time." This reviewer agrees that an exposition of the special theory of relativity based on such an idea is sorely needed and the author's "Ambition . . . to make space-time a real workshop for physicists, and not a museum visited occasionally with a feeling of awe" is laudable.

On the whole this is a well-written discussion of the following topics in special relativity: Kinematics, mechanics of single particles and systems of particles, mechanics of a continuum and electromagnetic theory. These topics are covered with varying degrees of thoroughness, completeness and quality of exposition.

The first chapter discusses the relationship between the metric of space-time and physical measurements, the latter being described in terms of ideal experiments. The author's intention is to lay a foundation strong enough to support both the special and general theories. This intention is fulfilled in a lively thought-provoking way.

The next four chapters are devoted to the geometry of flat space-time (Minkowski space), the group of this space (the Lorentz group), and the explanation of the classical experiments which were first satisfactorily accounted for by the Einstein special theory of relativity. The discussion of these topics is clarified greatly by using space-time diagrams in an effective manner.

Chapter IV which deals with the proper homogeneous Lorentz transformations which do not interchange the past and the future contains as one of its main theorems the incorrect statement: "Any finite Lorentz transformation (of the restricted class defined (above)) is equivalent to a 4-screw." A 4-screw is defined as "a rotation in a time-like 2-flat π, followed (or preceded by . . .) a rotation in a space-like 2-flat π^*, the 2-flats π and π^* being orthogonal to one another." The matrix representing a 4-screw may be taken to be