BOOK REVIEWS

Vector and tensor analysis. By Nathaniel Coburn. New York, Macmillan, 1955. 12+341 pp.

While unified under a single title and forming a coherent sequence, the material covered in this book could well be taught in three separate courses: an undergraduate course in vector analysis, and two graduate courses, one on tensor analysis and the other on elasticity and fluid dynamics.

The first four chapters of the book are devoted to vector analysis. The treatment is distinguished by a strong tensor flavor and by the fact that it is not confined to a single coordinate system. Throughout the book ample problems are provided at the end of nearly every section. As with most treatments of vector analysis, the sequence is from algebra to differentiation, integration and applications. A number of excellent applications to differential geometry are incorporated into the original exposition of vectors. The author's background in physics is evidently focused on mechanical problems and all other fields of physics are almost entirely ignored. When thermal problems are analyzed, they are viewed from a mechanical point of view, as on p. 79 where the author reasons, "If we assume that *heat behaves like a fluid mass and satisfies the continuity principle* . . . ." The modern physicist would reason simply, "If we assume that thermal energy is conserved. . . ."

Many topics are marked by their absence. No physical picture of the meaning of the divergence and curl of a vector is developed. Integral definitions of these concepts are not employed, resulting in a loss of physical clarity and in an unnecessarily lengthy development of the theorems of Stokes and Gauss. Other topics which are untouched are the scalar quasi-potential, the vector potential, classification of vector fields, and the entire range of fruitful applications to electromagnetic theory. The study of vectors concludes with a rather sketchy treatment of rigid-body motion and a rather beautiful and fairly detailed exposition of the flow of perfect fluids.

Tensors, on the other hand, are presented with a strong vector flavor. The approach is cautious. Vector notions are retained as long as possible and unnecessarily frequent use is made of unit vectors. First, tensors in orthogonal cartesian coordinates are treated. Thus, index notation is introduced without regard for the distinction between contravariance and covariance, and the coordinates of tensors are spoken of as components. Some may regard this backwards approach to tensors as pedagogically desirable; others will feel that the treatment of the trivial special case of tensors in orthogonal cartesian coordinates is an unfortunate method of introduction.

The subject is then extended to general cartesian coordinates. The transformations are centro-affine, but the geometry of affine space and its interesting physical applications are ignored. The useful distinction between space transformations and coordinate transformations is not made. The affine nature of the discussion that might have been expected here is quickly obviated by the introduction of the metric tensor. Even the metric tensor is not introduced in a form having general validity but as a method of calculating distances in the large by taking the scalar product of the radius vector with itself. Here, at long last, the student learns that "contravariant and covariant components" of a univalent tensor are not always equal.

The third chapter of the section on tensor analysis deals with tensors in general curvilinear coordinates. The conditions which an allowable coordinate system must satisfy are presented clearly. As the contravariant and covariant *coordinates* of univalent tensors have hitherto been referred to as *components* and as they obviously are not the same as the physically measurable vector components, a third kind of component is here introduced: the physical component. Though the metric tensor is not defined for general curvilinear coordinates, and the definition given in the previous chapter is no longer valid, the linear affine connection Γ^i_{jk} is defined in terms of unit vectors and the metric tensor. Subsequent topics are covariant derivatives, integrability conditions and nonholonomic coordinates.

By this time the reader may have gleaned a general idea of what is meant by a tensor, but nowhere is a definition of a tensor or of a geometric object given. Nor does the presentation clarify the fundamental question of which of the results of tensor analysis depend merely on the existence of a coordinatization of the points of space, which depend on the definition of Levi-Civita parallelism, and which require the introduction of a metric tensor.

A final chapter develops a formulation of the theorems of Gauss and Stokes in n-dimensional euclidean space, which is needed for the study of compressible fluids.

The crowning achievement of the book lies in the chapters on the application of tensors. The first of these chapters deals with the differential geometry of surfaces in euclidean 3-space. The treatment of elasticity which follows is particularly interesting in that it deals with finite deformations. Subsequent topics which are presented in a convincing manner in terms of tensors are viscous fluids, compressible fluids including the general theory of discontinuities and shock waves, and the theory of homogeneous statistical turbulence.

In the application of tensors to modern problems of fluid dynamics, the book is noteworthy. But a really satisfactory book on tensors should perform two functions. It should present tensors and related geometric concepts with clarity and precision. It should also give a well-rounded picture of the many fertile fields of application of tensors. Such a book remains unwritten.

DOMINA EBERLE SPENCER

Espaces vectoriels topologiques. (Chapters I-V, plus Fascicule de résultats). By N. Bourbaki. (Actualités Scientifiques et Industrielles, Nos. 1189, 1229, 1230.) Paris, Hermann, 1953–1955. 2+123+2 pp.; 2+191+3 pp., 2000 fr.; 2+39+1 pp., 400 fr.

Confronted by the task of appraising a book by N. Bourbaki, this reviewer feels as if he were required to climb the Nordwand of the Eiger. The presentation is austere and monolithic. The route is beset by scores of definitions, many of them apparently unmotivated. Always there are hordes of exercises to be worked through painfully. One must be prepared to make constant cross-references to the author's many other works. When the way grows treacherous and a nasty fall seems imminent, one thinks of the enormous learning and prestige of the author. One feels that Bourbaki *must* be right, and that one can only press onward, clinging to whatever minute rugosities the author provides and hoping to avoid a plunge into the abyss. Nevertheless, even a quite ordinary one-headed mortal may have notions of his own, and candor requires that they be set forth. We proceed, then, to a description of the present book.

Chapter I is entitled *Topological vector spaces over a field with a valuation*. It consists mostly of definitions and elementary theorems. As the coefficient field in later chapters is always the real or complex numbers, the emphasis here on arbitrary fields with valuation is hard to understand.

Chapter II deals with convex sets and locally convex spaces. It provides an excellent introduction to the subject. The Hahn-Banach theorem is given in several useful forms; Kreĭn's theorem on the extension of positive linear functionals is given, as well as the Kreĭn-Mil'man theorem. A curious appendix contains the Markov-Kakutani fixed point theorem (why not Schauder-Leray's or Tihonov's?), with an application showing the existence of an invariant mean for the con-