tinuous bounded functions on a topological Abelian group (a fact provable in other and simpler ways).

Chapter III takes up spaces of continuous linear mappings. The main theorem proved is the Banach-Steinhaus theorem, which appears as a statement about filters on the space $\mathcal{L}(E, F)$, where E is a "tonneau" space, F is a locally convex space with Hausdorff separation, and $\mathcal{L}(E, F)$ is all continuous linear mappings of E into F. In this formulation, the classical Banach-Steinhaus theorem and its elegant applications seem far away (although one standard application is given as an example). A host of types of topological vector spaces appear, mostly in exercises. Their utility for the general mathematician seems small.

Chapter IV, entitled *Duality in topological vector spaces*, is in the reviewer's opinion the most useful of all the five chapters. Here is a complete and readable account of the various topologies for the space of continuous linear functionals on a topological vector space.

Chapter V contains a treatment of the elementary theory of Hilbert spaces. Aside from a liberal use of filters, there seems to be little novelty in this chapter. One notes a surprising concession to human weakness—the author has furnished a couple of diagrams to illustrate a well-known theorem on the existence of unique distance-minimizing elements in convex sets.

In an interesting Historical Note, the author traces the history of the subject, from the contributions of D. Bernoulli to those of L. Schwartz. This spirited and at the same time learned account is well worth reading.

The "Fascicule de résultats" is of doubtful value. It would seem difficult to appreciate or use this brief summary without first having studied the main text: and when this has been done, the summary is not needed. A similar comment applies to the folded inserts at the ends of the volume repeating the most important definitions and axioms. A dictionary giving various common terms in English, French, and German is provided, as well as brief lists of special symbols.

To summarize: Plus ça change, plus c'est le même Bourbaki.

EDWIN HEWITT

Die innere Geometrie der konvexen Flächen. By A. D. Alexandrow Berlin, Akademie Verlag, 1955, 38.50 DM.

This is a German translation of the Russian book with the analogous title which appeared in 1948. Except for corrections of misprints and of some minor errors and the translator's simplifications of one or two proofs, the book differs from the Russian edition only by the addition of two appendices which report without proofs on recent developments. The author regrets that he could not integrate these into the text owing to other duties—he is rector of Leningrad University.

For this reason, and because a detailed description of the Russian edition is available in the *Mathematical Reviews* (vol. 10 (1949) pp. 619, 620), the principal results of the theory in its *present* stage will be briefly outlined here, marking results found in the appendices by (A).

H. Weyl put and partially solved the problem of realizing a prescribed line element of positive curvature on a sphere as a closed convex surface in E^3.[1] The central problem in A. D. Alexandrow's work is the analogue to Weyl's problem for perfectly general convex surfaces.

The intrinsic metric of a polyhedron in E^3 is locally euclidean at all points except the vertices. A neighborhood of a vertex is intrinsically isometric to the neighborhood of the apex of a cone. If the polyhedron is convex then the total angle of this cone at the apex (i.e. the angle after unfolding the cone) is less than 2π. We can therefore describe the intrinsic metric of a closed convex polyhedron as an intrinsic metric (i.e. one where distance equals the length of the shortest connection) on a sphere such that every point has a neighborhood isometric to a neighborhood of the apex of a cone with total angle less than or equal to 2π. The author's first fundamental result is that these trivially necessary conditions are also sufficient, i.e., that any metric on the sphere with these properties can be realized by one, and up to motions only one, convex polyhedron in E^3.

For the corresponding problem on general convex surfaces the simple characterization by conical neighborhoods no longer works. Any two points a, b on a closed convex surface S can be connected by a shortest curve whose length we call the distance of a and b. If $x(t)$ and $y(t)$, $0 \leqq t \leqq 1$, are two shortest arcs issuing from $p = x(0) = y(0)$ and $\alpha(t)$ is the angle at p' in the euclidean triangle $p'x_t'y_t'$ for which the lengths of the sides equal the corresponding distances of p, $x(t)$, $y(t)$ on S, then $\alpha(t)$ is a nonincreasing function of t. This

[1] *Über die Bestimmung einer geschlossenen Fläche durch ihr Linienelement*, Vierteljahrschrift der naturf. Ges. Zürich vol. 61 (1916) pp. 40–72. The problem was first solved completely in the analytic case by H. Lewy in: *On the existence of a closed convex surface realizing a given Riemannian metric*, Proc. Nat. Acad. Sci. U.S.A. vol. 24 (1938) pp. 104–106.

important fact is not easy to prove and is called the convexity condition. Thus the intrinsic metric of a closed convex surface may be described as an intrinsic (see above) metrization of the sphere which satisfies the convexity condition. Any metric on a sphere with these properties can be realized by one, and up to motions only one, closed convex surface in E^3. The author proves the existence by approximation with polyhedra, the uniqueness is, of course, equivalent to the rigidity of general convex surfaces and is due to Pogorelov (A). Because the convexity condition is not obvious and also in some respects too strong, it is shown to be equivalent to another condition which requires that, with a properly defined angle, the excess (i.e., the sum of the angles minus π) in a small geodesic triangle is non-negative.

This result is combined with another essential idea which the author calls the method of gluing. Although it applies to general convex surfaces, we formulate it here only for pieces of smooth surfaces, because this will bring out the idea more clearly. Assume we have a finite number of surfaces of non-negative curvature each homeomorphic to a disk and bounded by a finite number of smooth arcs. Assume further that boundary arcs are abstractly identified such that (1) the resulting surface is, topologically, a sphere, (2) any two identified (partial) arcs have the same length (3) at any point lying on exactly two boundary arcs the sum of the geodesic curvatures of these arcs (towards the domains they bound) is non-negative (4) the total angle at a point belonging to more than two domains is at most 2π. Then this abstract surface can be realized as a convex surface in E^3.

Let an intrinsic metrization of the plane as a complete space be given which satisfies the convexity or the excess condition. By the method of gluing the author constructs a sequence of closed surfaces containing larger and larger pieces of the given metric and thus proves that the given metric can be realized by a complete unbounded convex surface in E^3. The realization is unique if the integral curvature of the given metric equals 2π (Pogorelov (A) under certain additional hypotheses, and recently for the general case in Doklady Akad. Nauk U.S.S.R. vol. 106 (1956) pp. 19–20). It is not unique when the integral curvature is less than 2π (Oloyanišnikov (A)). The author also uses the method of gluing to realize partial surfaces, i.e., intrinsic metrics satisfying the excess condition and defined on connected open sets of the sphere. The realization is, of course, in general not unique.

These results changed Weyl's original problem entirely: the restriction to closed surfaces became unnatural, the existence of a realization was no longer a problem, but there emerged the novel problem of proving the smoothness of *all* realizations provided the given metric is smooth. This was accomplished by Pogorelov (A) in all cases under the assumption that the coefficients E, F, G have locally bounded fifth derivatives and that the curvature is positive. The, in some respects, most interesting cases, where E, F, G are of class C^2 or C^3 (the importance of this case derives from the Gauss-Codazzi Equations) remain open.

A simple example will exhibit the strength of these results. Consider a cap K_0 of the unit sphere with integral curvature less than 2π and bounded by a circle C_0. Imbed C_0 in any continuous family C_t of plane convex curves with the same length bounding domains D_t. By the gluing theorem the abstract surface resulting through identification of the boundaries C_t of D_t and C_0 of K_0 can be realized as a convex surface K_t' in E^3. Pogorelov's results guarantee that K_t' is unique (up to motions) and that the interior K_t of the part K_t' corresponding to K_0 is analytic. Thus we obtain a continuous deformation of K_0 through analytic surfaces.

The author uses his methods to prove analogous realization theorems in spaces of constant positive and negative curvature. The latter case is by far the more interesting because a convex surface in hyperbolic space may have negative Gauss curvature and is, topologically, any open connected subset of the sphere. An abstract surface of this general topological type whose curvature is bounded from below can be realized as a convex surface in a hyperbolic space.

This review of results does not give any indication about the methods. Most concepts, in particular that of angle and total geodesic curvature of an arc, require a detailed study with laborious proofs before they become applicable to the general situations treated by the author. In collaboration with some of his students the author showed (A) that these concepts are applicable to a wide class of surfaces, namely the surfaces of bounded curvature (A). They include all surfaces which can be uniformly approximated by sequences of smooth surfaces with uniformly bounded integral curvatures; they comprise, in particular, all polyhedra with a finite number of vertices. This fact alone should suffice to make the theory appealing to any true geometer.

In spite of some very long proofs the book is very easy to read, because the author spares no trouble in outlining his aims and procedure beforehand, and has a flair for setting the reader at ease. The translation, which is excellent throughout, is especially successful in reproducing this effect.

HERBERT BUSEMANN