field and then defines these quantities as limits of integrals. The study of the del operator and Laplacian and Newtonian fields concludes this section of the text.

Vectors and tensors in affine and metric n-dimensional space are treated in Chapter IV. First, the author discusses the curl of a vector and the divergence of a vector density in affine n-space. This is followed by a discussion in metric space of the metric tensor, the absolute derivative, geodesics, and related topics.

Finally, the author considers the following topics in electromagnetic theory: electrostatic fields (force, energy, and polarization); electric currents (steady and non-steady) and the laws of Kirchoff, Joule, and Ampère; magnetic and electromagnetic fields. The last topic is presented with considerable skill and ranges from the Maxwell laws of classical three-dimensional Euclidean space (presented by classical vector methods) to the Lorentz-Einstein transformation in Minkowski space and the Maxwell tensor of relativity.

The level of the text is such that a mathematically mature student with a background in classical physics can follow the developments. The book should be of particular interest to physicists and engineers.

N. COBURN

Surface area. By L. Cesari. (Annals of Mathematics Studies, no. 35.) Princeton University Press, 1956. 10+595 pp. $8.50.

The length $l(C)$ of a curve C is the limit of the lengths $l(p_n)$ of inscribed polygons $p_1, \cdots, p_n, \cdots$ such that the maximum side-length of p_n converges to zero as $n \to \infty$. Almost eighty years ago, Schwarz and Peano noted that the analogous statement for surface area (in terms of the elementary areas of inscribed polyhedra) is false even in the simple case when the surface under consideration is the lateral surface of a circular cylinder. Subsequently, many other phenomena were noted which revealed further fundamental discrepancies between arc length and surface area. The immediate issue raised by the initial observations of Schwarz and Peano was, however, the formulation of a logically consistent definition of surface area. During the past eighty years, many such definitions have been proposed, and an enormous amount of effort has been expended in the study of these various concepts of surface area. As far as mathematical fields of a classical type are concerned, the reassuring inference from these studies is the fact that in reasonably decent cases the classical integral formula taught in calculus does indeed yield the correct value of surface area. On the other hand, the need for a comprehensive general theory of surface area became apparent several decades ago in various modern fields of Analysis and Geometry.

Thus surface area theory developed partly in response to needs arising in various mathematical fields, and partly in response to the challenge presented by apparent discrepancies between arc length and surface area. Furthermore, the confusion created by the extreme diversity of the definitions proposed for surface area constituted by itself a strong challenge to develop a theory of compelling unity, beauty, utility, and generality. The concept of surface area proposed by Lebesgue around the turn of the century (to be referred to as the Lebesgue area) has been adopted for this purpose by many workers in this field. In its present highly developed form, the theory of the Lebesgue area possesses several gratifying features. It reveals that the discrepancies between arc length and surface area (referred to above) are compensated for by a number of far-reaching analogies. Also, the theory shows that many of the conflicting definitions proposed for surface area become equivalent with the Lebesgue area provided that the underlying intuitive geometrical ideas are phrased in a reasonably relaxed manner. Furthermore, the theory yields important applications in relation to various classical and modern problems in other fields. Professor Cesari, the author of the book under review, made many contributions of the highest importance in surface area theory and in related fields, and therefore the publication of his present book is a significant event. For the general mathematical community, the book brings a beautifully organized presentation of many of the fundamental accomplishments in the theory of the Lebesgue area, as well as a wealth of informative and penetrating comments on historical background and motivation. For the specialist in surface area theory and in related fields, the book of Cesari is an indispensable library item, with many features of compelling interest. One finds here the first exposition in book form of the famous *Cesari inequality* which states that the Lebesgue area of a surface is less than or equal to the sum of the Lebesgue areas of its orthogonal projections (considered as *flat surfaces*) upon the three coordinate planes. From the intuitive point of view, this inequality is merely an obvious property of surface area, and yet the actual proof of the inequality by Cesari has been justly hailed as one of the most striking achievements in surface area theory. Indeed, the Cesari inequality furnished the missing link in establishing in full generality several fundamental theorems, and furthermore, its proof contained auxiliary results of a striking and novel character. One of these, the so-called *four-line theorem*, is concerned with a remarkable homotopy property of Euclidean 3-space. Subsequently, Cesari showed how to avoid the use of this theorem by ingenious analytic devices. Both of these alternative approaches are treated fully in the book. One finds here also the first exposition in book form of the fundamental *Cesari representation theorem*, which states that every surface (this term being taken in the appropriate precise meaning explained in the book) of finite Lebesgue area admits of a conformal representation (in a certain generalized sense) such that in terms of this representation the Lebesgue area is given by the classical integral formula. An important feature of the book is the generality of the mappings which occur as representations of surfaces. Indeed, Cesari operates with mappings from plane sets of a general type which include open sets and finitely connected Jordan regions. From the point of view of general methodology, the following feature of the book is of special interest. Cesari notes that several of the fundamental theorems of the theory are concerned with a given individual representation of the surface, and he suggests that in such cases the proofs should be made in terms of the given representation, without using the existence of more favorable representations for the same surface. In carrying out this program throughout the book, he devises novel and most instructive proofs for several fundamental theorems and develops remarkable new methods. This feature of the book should give much food for thought even to the most seasoned specialist in the field. To the reader interested in applications, the Appendix on the Weierstrass-type integral developed by Cesari should be especially valuable.

The presentation is careful, precise, and concise throughout. Both for reference and for study, the book is an invaluable addition to the literature on surface area theory.

T. RADO

Additive Zahlentheorie. Part I. *Allgemeine Untersuchungen.* By Hans-Heinrich Ostmann. (Ergebnisse der Mathematik und ihrer Grenzgebiete, New series, no. 7.) Berlin, Springer, 1956. 7+233 pp. 29.80 DM.

The author has chosen to present additive number theory from the point of view of the theory of density of sets of integers. The largest part of this volume is devoted to the development of this theory and closely related topics, but certain other parts of additive number theory are included.

The basic concepts are the system Σ of sets $\mathfrak{A}$ of non-negative integers and the operation $\mathfrak{A}+\mathfrak{B}=\{a+b;a\in\mathfrak{A},b\in\mathfrak{B}\}$. Their proper-