In both Hildebrand and Kopal there seems to be an overemphasis on approximate quadratures. Kopal has interesting historical notes, a detailed account of the choice of interval in numerical differentiation and some discussion of boundary value problems (in the one-dimensional case) which was not readily accessible in English. The mathematician reading Kopal will be disturbed from time to time by such phrases as "In general, we may expect . . . ," which he can, often, rightly question.

Certainly the second two books will be of value to the teacher and research worker, but they seem to be far too extensive for the ordinary student, whom they may discourage. There is no doubt that there are many principles in numerical analysis which can be incorporated at various places in regular courses and teachers can find suitable material in all three volumes. For instance, an efficient method for solving linear equations, with checking devices, can be discussed at the beginning of algebra courses. The concept of differences and Lagrangian ideas can be introduced when polynomials are being studied. Then, when calculus is being started, the ideas of numerical differentiation and integration can be added. When differential operators are being discussed in connection with differential equations with constant coefficients, an account of difference operations can be added. There are many opportunities in courses on matrix theory, to introduce ideas useful in numerical analysis. In analytical geometry, too, the extremal properties of the principal axes of conics leads to the ideas of Rayleigh, Ritz, and others. All this can be covered with the use of a desk calculator or two, and a few books of tables as a source for examples. These principles could be later consolidated in a short course, and there would seem to be a need for an appropriate text, of a fraction of the size of any of the three. A shortened version of Hildebrand could be very useful. This would be supplemented in due course by a similar account of the principles of programming—as soon as these are established—and of what we can call "modern" numerical analysis, the type of numerical analysis relevant in the use of the high speed automatic computers.

John Todd

Algèbre. By R. Ballieu and F. Simonart. Louvain, Librarie Universitaire and Paris, Gauthier-Villars, 1955. xvi+355+1 pages.

In this book, classical algebra is treated, to quote the senior author, Simonart, "en langage moderne." The central themes are those aspects of linear algebra which pertain most directly to the solution of simultaneous linear equations over a field, and the study of polynomials and their roots, the theory of equations. This second theme is developed in the best expository tradition of the field. One cannot, unfortunately, say the same for the first. In a preliminary chapter, containing introductory material on such topics as groups, rings, fields and formal polynomials, matrices are introduced as arrays, and matric representations are given for both the complex numbers and the quaternions. Chapter II on vector spaces and linear systems, Chapter III on determinants, Chapter IV on simultaneous linear equations and Chapter V on quadratic forms contain, in addition to the broad subjects of their titles, such diverse topics as orthogonal matrices, centers of polynomials and Vandermonde's determinant. Many of the sections in this portion of the book, for instance, §18 on the evaluation of determinants and §27 on invariants and covariants, are very well written. The disappointing feature of the first five chapters is that the authors, in places, manage to leave the impression that the introduction of modern language into this old field is really unnecessary. The core of linear algebra, the linear transformation, enters, as it should, repeatedly, but its basic property of linearity plays only a minor role in this exposition, if it is ever really mentioned. Another omission of this type occurs in §18, 5. Here, the characteristic polynomial is introduced by fiat, and all connections with vital aspects of linear transformations are suppressed. Likewise in §9, 4, equivalence and similarity of matrices are defined with almost no indication that these concepts involve change of bases in suitable vector spaces.

Chapter VI on polynomials covers standard material on roots, interpolation, resultants and multiplicities. The treatment of the resultant is particularly comprehensive, but one could wish that its introduction had been better motivated. In §31, 2, Sylvester's determinant appears without explanation, and it becomes clear only later why this particular determinant is the key to the theory. The discriminant appears in the next chapter on symmetric functions.

The last two chapters are easily the best of the book. In Chapter VIII on transformations of polynomials, the effect of the rational substitution $y=P(x)/Q(x)$ on the polynomial $f(x)$ is studied by forming the resultant of f with $yQ-P$. The transformations are simplified in steps so that, as an end result, the method of diminishing the roots of f by a fixed constant arises naturally. Considerations of polynomials in which the roots are bound to each other by algebraic equations lead to a long section on the reciprocal equation. Wherever convenient, the authors apply the results of the chapter on symmetric functions to this development. Binomial equations come next with just enough group theory to organize the material. For instance, the standard properties of the Euler totient appear here as group-theoretic corollaries. The chapter ends with an adequate treatment of the cubic and quartic. The last chapter on the location of roots, both real and complex, is especially detailed. Convergence questions for both Newton's process and for the regula falsi are treated. Contemporary results (with references to live authors) are included in §48 where zeros of complex polynomials are discussed in such a way that the reader is not left with the all too common impression that the subject is embalmed. Although there are no exercises, the text abounds with well chosen examples. The few errors which appear seem to be caught in the errata at the end.

FRANKLIN HAIMO

Handbuch der Laplace-Transformation, Vol. II. *Anwendungen der Laplace-Transformation*. Part 1. By Gustav Doetsch. Birkhäuser, Basel and Stuttgart, 1955. 434 pp. 56.15 Swiss francs.

In this volume the author covers a wide range of applications, many of which might be considered as being applications of the Laplace transform, rather than properties of the Laplace transform, only by fiat. He begins with a collection of results from the first volume (cf. Bull. Amer. Math. Soc. vol. 58 (1952) p. 670), the "rules" for operating with the Laplace transform. Then he takes up a wide variety of connections between the asymptotic expansions of a pair of transforms. Ordinary Abelian and Tauberian theorems were dealt with in the first volume; here it is a question of what can be deduced about one member of a pair of transforms when an asymptotic expansion for the other is given. Much of this material is discussed here for the first time in systematic form, and many old isolated results now appear as special cases of the general theory. Numerous illustrative examples are taken up, for example Stirling's series for log $\Gamma(s)$, Bessel functions, theta functions, and the wave function for the hydrogen atom.

The second part of the volume takes up the connection between Laplace transforms and factorial series (which the author thinks deserve more attention than they have been getting). A number of miscellaneous convergent expansions are also discussed.

The third part deals exhaustively with the use of the Laplace transform to solve ordinary differential equations, first those with constant coefficients on a half line, then those with constant coefficients on a whole line, and finally those with variable coefficients, in so far as the method is applicable to them. Problems about servomecha-