with relative admissibility (§7), two major though probably corrigible errors occur. First, as H. B. Curry discovered, the rule $a,\ b \rightarrow a \wedge b$ is not relative admissible, e.g., in the system whose only rules are $+$ and $+a \rightarrow 0$. Second, $a \wedge b \rightarrow a$ is not admissible, e.g., in the system whose only rules are $+$ and $+a+ \rightarrow ++a+$ and (D_1^0). Despite all these shortcomings, which probably require a thorough revision to overcome, the book constitutes an essentially sound and indeed outstanding contribution.

WILLIAM CRAIG

Introduction to Mathematical Logic, Vol. I, by Alonzo Church. Princeton, The Princeton University Press, 1956. 10+376 pp. $7.50.

This is a revised edition of the slim, paper-backed volume which appeared in 1944 as one of the Annals of Mathematics Studies. Of the five chapters two are devoted to propositional calculus, two to functional calculi of first order, and the last to second-order calculi, so that the plan of the original edition is followed to treat the most basic formal systems of mathematical logic. But the material of the original has been so greatly altered and expanded that it seems best to report the important features of the new work directly, rather than to compare it in detail with its predecessor.

Several unusual features of the book are apparent even before one begins to read: There is an introduction 68 pages in length; it is divided into 10 sections, which is exactly the number of sections in each of the five chapters; there are 590 consecutively numbered footnotes, at least one of which is a full page in length; there was a lapse of five years between the completion of the writing and the appearance of the published volume. While there are separate sections for historical notes, these spill over liberally into the highly informative footnotes; so that when the author's pre-eminent reputation for painstaking attention to historical detail is considered, it seems manifest that this work will quickly establish itself as a definitive reference volume. The book also contains a great many exercises, ranging from simple illustrations to brief sketches of developments not treated in the text. For some curious reason textbooks in symbolic logic have always evinced a conspicuous paucity of problems suitably challenging to mathematically inclined students, a phenomenon which has tended to place this subject at a competitive disadvantage with most of the other mathematical sciences. The present innovation is thus very welcome, and will greatly enhance the value of the book's use in connection with beginning courses for students with some background of mathematical experience.

The long introduction is essentially a conceptual framework for a general theory of linguistic systems, important features of which, especially the treatment of the concepts of sense and denotation (which are fundamental to Church's analysis of language), the author traces back to Frege. From the strict viewpoint of pure mathematics it is possible to regard the technical development of the subject which is given in the five chapters as quite independent of the material contained in the introduction; but the emphasis throughout the book is on logic as *applied*, rather than pure, mathematics, and from this viewpoint the introductory material becomes relevant as explaining the relation between the mathematical theory of formal systems and the languages (of which these systems are idealized models) to which the theory applies. The linguistic analysis which the introduction affords is the subject of continuing controversy in philosophical circles, but this in no way reflects on the mathematical portions of the book. Despite its non-mathematical treatment the introduction holds much of interest for the working mathematician, as it illuminates many basic concepts applicable to mathematical language, concerning which vague and often erroneous ideas are widespread.

In defining "the logistic method" (p. 47) the author explains how some natural language, such as English, may be used as a metalanguage for describing and referring to a formalized object language. However, he emphasizes the desirability of restricting the use of English to a portion of that language which is "just sufficient to enable us to give general directions for the manipulation of concrete physical objects . . . "; in particular, "those additional portions of English are excluded which would be used in order to treat of infinite classes . . . ". It seems to this reviewer that these restrictions are not strictly observed in the author's practice, and that the question of whether they can in principle be followed is open to doubt. For example, at the beginning of the description of the first logistic system (p. 70) the definition of "well-formed formula" is given by stating three formation rules and then adding: "To complete the definition we add that a formula is well-formed if and only if its being so follows from the three formation rules. In other words, the well-formed formulas are the least class of formulas which contains all the formulas stated in 10i and 10ii and is closed under the operation of 10iii." The mention of this least class of formulas, or of the unspecified notion "follows from," seem rather clearly to be couched in portions of the English language which were proscribed.

The first chapter treats in detail a particular system of propositional calculus, including such standard topics as the deduction theorem, decision problem, duality, consistency, completeness, and independence. In the second chapter other complete systems of propositional calculus are considered more briefly, as well as partial systems (including the intuitionistic) and extended systems. Altogether 40 different systems are described, and each is given a name consisting of the letter "P" with sub- or super-script adjoined. This proliferation reflects the state of the literature of the subject, and has desirable features both from the points of view of textbook and reference work; but an untoward effect is to leave the unaided beginning reader confused by the sheer mass of detail. The mathematician who is seeking a first knowledge of technical logic in this book should be warned that the practice of this subject does not consist simply in the performance of endless (or even forty) variations on a theme.

Many systems of first-order functional calculi are considered in Chapters III and IV. The substitution rules are studied in detail, the prenex and Skolem normal forms are treated, as well as more advanced topics such as Gödel's completeness theorem, the Skolem-Löwenheim result, and the decision problem (solution of special cases, reductions of the general problem). The use of particular first-order calculi as systems for formalizing branches of mathematics, such as number-theory or geometry, by the addition of formal axioms which are not valid in arbitrary domains, is put off to section 55 (Postulate theory), which is imbedded in Chapter V (Functional calculi of second order); the fruitful mathematical applications of the completeness theorem to these systems, which have recently been obtained by several authors, are not mentioned. In this connection the reviewer regrets that Church did not see fit to extend the notion of first-order functional calculus so as to include systems which contain operation symbols in addition to functional (i.e., predicate) symbols. Such systems are particularly natural and useful in formalizing many mathematical theories, and the principal metamathematical results, such as Gödel's completeness theorem, apply to them equally. It is true that operation symbols are in principle dispensable when predicate symbols are available, but this fact is itself worthy of study in a comprehensive treatment such as the present volume, and certainly does not warrant ignoring a kind of symbol so widely used in mathematics.

In the final chapter on second-order calculi attention is principally bestowed on the *simple* calculi which correspond to formalized systems of classical mathematics; but there are also two sections devoted to the predicative and ramified calculi which were devised to meet criticisms developed as an outgrowth of "constructive" view-points in mathematics. For the simple calculi the reviewer's completeness theorem is established and then the addition of formal axioms of infinity, and of well-ordering, is considered.

At various points in the book reference is made to topics which are to appear in a projected second volume. A tentative table of contents for this volume is given, listing chapters on higher-order functional calculi, second-order arithmetic, Gödel's incompleteness theorems, recursive arithmetic, an alternative formulation of the simple theory of types, axiomatic set-theory, and mathematical intuitionism. The appearance of this volume promises to complete a work of great usefulness both for students and scholars, and it is to be hoped that a way can be found to shorten the publication time.

LEON HENKIN

Fundamental Concepts of Higher Algebra, by A. Adrian Albert, University of Chicago Press, 1956. 9+165 pp.

Finite fields are soiled: computing machines are beginning to use them. Dickson's *Linear groups and the Galois theory* is the classical exposition of the subject, but since it was written modern algebra has come into existence; Albert's avowed purpose is therefore to give us a timely, modern version of the theory, setting it within its proper context as part of general field theory. But books behave waywardly while they are being written; in a sort of Tristram Shandy fashion four-fifths of this one is over before we get around to the finite fields, and then what we do read about turns out to be a rather brief and odd assortment of material—perhaps representing what will be most useful to those with practical applications in mind. On the other hand, the main part of the book consists of a good, compact presentation of the essentials of modern algebra. As such, it has a wide potential audience and has in fact been written with a wide class of readers in mind. In short: a nice, lightweight algebra text with an addendum on finite fields.

A great deal is covered in its 150-odd pages. The first two chapters discuss in turn group theory (no operators) through the Jordan-Hölder theorem and basis theorem for abelian groups, then elementary ring and ideal theory, with a discussion of factorization for polynomials in one variable. Chapter three treats vector spaces and matrices; elementary transformations of matrices, the characteristic function, and elementary divisor theory are discussed in some detail and there is in the problems a heavy classical emphasis on matricial computations: triangularization, determination of rank, inverse, and invariant factors. The fourth chapter deals with field extensions and