

points in mathematics. For the simple calculi the reviewer's completeness theorem is established and then the addition of formal axioms of infinity, and of well-ordering, is considered.

At various points in the book reference is made to topics which are to appear in a projected second volume. A tentative table of contents for this volume is given, listing chapters on higher-order functional calculi, second-order arithmetic, Gödel's incompleteness theorems, recursive arithmetic, an alternative formulation of the simple theory of types, axiomatic set-theory, and mathematical intuitionism. The appearance of this volume promises to complete a work of great usefulness both for students and scholars, and it is to be hoped that a way can be found to shorten the publication time.

LEON HENKIN

Fundamental Concepts of Higher Algebra, by A. Adrian Albert, University of Chicago Press, 1956. 9+165 pp.

Finite fields are soiled: computing machines are beginning to use them. Dickson's *Linear groups and the Galois theory* is the classical exposition of the subject, but since it was written modern algebra has come into existence; Albert's avowed purpose is therefore to give us a timely, modern version of the theory, setting it within its proper context as part of general field theory. But books behave waywardly while they are being written; in a sort of Tristram Shandy fashion four-fifths of this one is over before we get around to the finite fields, and then what we do read about turns out to be a rather brief and odd assortment of material—perhaps representing what will be most useful to those with practical applications in mind. On the other hand, the main part of the book consists of a good, compact presentation of the essentials of modern algebra. As such, it has a wide potential audience and has in fact been written with a wide class of readers in mind. In short: a nice, lightweight algebra text with an addendum on finite fields.

A great deal is covered in its 150-odd pages. The first two chapters discuss in turn group theory (no operators) through the Jordan-Hölder theorem and basis theorem for abelian groups, then elementary ring and ideal theory, with a discussion of factorization for polynomials in one variable. Chapter three treats vector spaces and matrices; elementary transformations of matrices, the characteristic function, and elementary divisor theory are discussed in some detail and there is in the problems a heavy classical emphasis on matricial computations: triangularization, determination of rank, inverse, and invariant factors. The fourth chapter deals with field extensions and

Galois theory, concluding with the normal basis theorem. The detailed study of particular fields is the only way to learn Galois theory; all algebra students should be directed to the more than two pages of specific computational problems about fields of all sorts that Albert includes here.

After deriving the basic elementary results, the bulk of the final chapter on finite fields is devoted to a somewhat heterogeneous collection of techniques for finding irreducible polynomials. A formula of Dedekind for the product of the irreducible polynomials of degree n over $GF(q)$ is proved; a similar but more refined formula of Dickson is stated later with an indication of proof and used as the basis of a computational procedure for determining all the polynomials. For certain not-too-special cases, Albert obtains more explicit determinations by proving theorems he attributes to Serret, these resting ultimately on the following simple lemma: if $f(x)$ is of degree m and irreducible over $GF(q)$, with roots of multiplicative period e , then $f(x^r)$ will also be irreducible, where r is any prime such that $r|e$, $r \nmid (q^m - 1)e^{-1}$. Many specific computations using the methods are given, and again, there are plenty of problems. The chapter concludes in a curious fashion by stating without proof theorems of Dickson on a variety of subjects: specific equations of low degree, substitution polynomials, irreducibility criteria. Apparently, this section is meant as a summary (partial?) of Dickson's work in finite fields. If so, then first this should have been stated, and second, the assembling of results should not have been done in such an obviously hasty and disorganized manner—Dickson deserves better. But if it was meant to give one some idea of the modern theory of finite fields, then the omission of a result of such generality as the Riemann hypothesis is well-nigh incomprehensible. A bibliography and tables of least primitive roots and irreducible polynomials are appended.

The universal aims of this book place it more or less inevitably in the please-all-of-them-some-of-the-time category: everyone is offered a berth, but no one will find the fit quite exact. Those wanting to learn or review algebra will be happy to find that in remarkably few pages all the essentials are treated, and they will (or ought to be) delighted with the problems, which will help self-study greatly. But in the condensation the leisurely inter- and intra-theorem explanations of, say, the author's *Modern higher algebra* have had to be sacrificed, and the proofs have become little pieces of old hen: nourishing, but tough and chewy. Albert is well aware of this, of course; he has elected to write his book in a certain style and in so doing he is not without plenty of distinguished predecessors. My own feeling is that

it is a poor style for a textbook—especially one for the generally undisciplined American. If the book accompanies a course or is being used for review—his own suggestions for its use—then this is perhaps not such a very serious matter, and after all, the theorems do in a sense speak for themselves. I think however that most students welcome toastmasters who are willing to say a little more than the inevitable and perfectly useless cant, “We have the following result . . . we next prove”

Those interested in finite fields will find the last chapter interesting and presumably useful; on the other hand they might wish the preceding exposition trimmed of the material which is not really relevant (all the matrix theory, for instance) and to see the extra space used to expose the finite fields in a more complete, more organized, and more explanatory fashion. The computers perhaps have no use for it, but I for one was pained by the omission of Chevalley's theorem that a finite field is quasi-algebraically closed (“Demonstration d'une hypothèse de M. Artin,” which Albert points out was actually conjectured some 25 years earlier by Dickson—but Artin's reasons were elegant and convincing). It is a theorem which goes significantly into the structure of the finite field, and at the same time it has one of the most beautiful proofs in algebra. It seems that the machines are to crush our daisies, too.

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BRIEF MENTION

Einführung in die höhere Mathematik. vol. 1. By H. v. Mangoldt and K. Knopp. 10th ed. Stuttgart, S. Hirzel, 1956. 16+564 pp. DM 23.

A new edition of this textbook of basic analysis.

Cours de mathématiques. By J. Bass. Paris, Masson, 1956. 11+916 pp. 7800 fr. (paper), 8500 fr. (cloth).

A textbook of analysis through partial differential equations, intended primarily for engineering students.

A dictionary of statistical terms. By M. G. Kendall and W. R. Buckland. New York, Hafner, 1957. 11+493 pp. \$4.50.

Prepared at the invitation of the International Statistical Institute, this book includes also glossaries of statistical terms in French, German, Italian and Spanish.

Theory of Lie groups I. By C. Chevalley. Princeton University Press, 1946. 232 pp. \$2.75 (paper).