shown, with considerably more difficulty, that in the case $\alpha_1 = \cdots = \alpha_n = 0$, the additional hypothesis that ψ is monotone decreasing guarantees that the inequalities in question have infinitely many solutions for almost no or almost all sets $(\theta_1, \cdots, \theta_n)$, according as the above series converges or diverges. This hypothesis is weaker than that in the similar theorem of Khintchine.

In the final chapter the Pisot-Vijayaraghavan (PV) numbers are studied. These are the algebraic integers $\alpha > 1$ all of whose conjugates except α itself lie in the open disk $|z| < 1$. It is easy to see, by considering the trace of α^n, that $\|\alpha^n\|$ approaches zero as $n \to \infty$ if α is a PV number. Pisot showed, conversely, that if $\alpha > 1$ is algebraic and $\lambda \neq 0$ is real, and if $\|\lambda\alpha^n\| \to 0$, then α is a PV number. Moreover, he showed that if $\alpha > 1$ is real, and if $\sum \|\lambda\alpha^n\|^2 < \infty$, then α is algebraic and therefore a PV number. (It is an open question whether $\|\lambda\alpha^n\| \to 0$ implies that α is algebraic.) Salem obtained the unexpected result that the set of PV numbers is closed. Proofs are given here of these three theorems; they are much simpler than the original proofs, although similar in conception.

The book closes with three appendices giving necessary tools from linear algebra and geometry of numbers, and a bibliography of papers mentioned.

W. J. LeVeque

Irrational numbers. By Ivan Niven. Carus Monograph no. 11: New York, Wiley, 1956. xii+164. $3.00.

This most recent in a series of distinguished monographs is outstanding in organization, in clarity, and in choice of material. The book, which begins with "the preponderance of irrationals," and which closes, in Chapter X, with a proof of the Gelfond-Schneider theorem, is an admirable fulfillment of the author's purpose: "an exposition of some central results on irrational numbers . . . the main emphasis [being] on those aspects . . . commonly associated with number theory and Diophantine approximations."

The topics are arranged, in general, in order of difficulty, with the result that some of the theorems in the early part of the book are subsumed under stronger theorems later. This organization seems to have real pedagogical value. The same sort of organization is followed, to some extent, within each chapter; for example, a theorem on the uniform distribution of a sequence of irrationals is first proved by use of results on continued fractions (one of the few cases where appeal is made to the material of an earlier chapter), and then obtained again as a corollary of Weyl's method of trigonometric sums (the needed theorem about a Fourier series being stated and reference given for its proof).

In addition to the topics already mentioned, and to the basic subjects which would be expected (algebraic and transcendental numbers, approximation of irrationals by rationals), there are two lengthy chapters, one on Normal Numbers and another on The Generalized Lindemann Theorem. At the end of each chapter there is a set of Notes elaborating the material. A List of Notation and a Glossary supplement the Index.

Perhaps 164 pages is a natural limit for a monograph of this type, but it seems a shame that the book isn't 20 pages longer. The addition would permit the amplification of the all-too-brief Notes (as it is, Roth's result on the approximation of algebraic numbers is passed off in two sentences) and the incorporation of exercises. True, the interested reader is given ample references for further study, but the book would be much more nearly self-contained if it were slightly longer.

It may be appropriate in this review to remark that Niven smooths the way for his readers by avoiding two difficulties:

(1) *The Roadblock.* It is not uncommon to come upon a statement in print and to ask oneself, "How does he know *that* ?" The obedient and obstinate reader stops short and does his best to justify the puzzling statement, convinced that to do so should be simple—for hasn't the author thought it so obvious that he hasn't considered an explanation necessary? Reading on, one discovers that the author devotes the next page and a half to the justification. If every author would develop a style which makes a clear distinction between statements which are "obvious" and those which he proposes to prove in the sequel, readers would have an easier time.

(2) *The Hill-and-Valley.* A reader sometimes finds that he needs help in filling gaps in an exposition. Then he may ask, "Why did the author leave this to me, while, three pages back, he gave all the details of an argument that I found trivially simple without his help?" The problem of uniformity or evenness of exposition is essentially unsolvable, because of the well-known fact that one man's Mede is another man's Persian; but the conscientious expositor can at least try out his presentation on several victims before publishing it.

It is a pleasure to report that Niven has done a masterful job of exposition.

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