BOOK REVIEWS

Intégration (Chapter V). By N. Bourbaki. Actualités Scientifiques et Industrielles, no 1244, Paris, Hermann, 1956. 2+131 pp. 1600 fr.

This is the second of an as yet unannounced number of fascicles of Bourbaki's work on integration. The preceding fascicle, containing Chapters I–IV, was reviewed in this *Bulletin* vol. 59 (1953) pp. 249–255.

The title of Chapter V is *Intégration des mesures*. This refers to the following maneuver. Let X and T be locally compact spaces; let μ be a positive measure on T; for each $t\in T$, let λ_t be a positive measure on X. Subject to appropriate conditions on the mapping $t\rightarrow\lambda_t$, it follows that $\nu=\int\lambda_t d\mu(t)$ is a positive measure on X. Various specializations and applications of this idea lead to a number of important topics. Among the major items considered are the following.

(1) Let π be a mapping of T onto X; let g be a non-negative function on T; and let $\lambda_t=g(t)\epsilon_{\pi(t)}$ where ϵ_x is the measure generated by a unit mass point at x. This leads to topics in the theory of discrete measures.

(2) Let $X=T$, and let π be the identity map; then formally, $\nu=\int g d\mu$, and the Radon-Nikodym theorem emerges.

(3) Let $g(t)\equiv 1$; then ν is the image under π of μ, and the discussion turns to changes of variable in an integral.

(4) Integration with respect to ν generates an iterated integral, and this leads to the Fubini theorem.

This brief outline indicates roughly the content of the chapter and also the extremely elegant way in which the entire discussion is centered around the idea of integration of measures. Regarding the work as an isolated essay on selected topics, one cannot praise it too highly. The treatment is beautifully unified and smooth to the point of seeming effortless. It is a delight to read.

Unfortunately, when this reviewer tries to place the work in the literature of measure theory, he is led to some unfavorable reactions. For one thing, the authors relegate to very minor roles a number of items that many people consider important. Examples:

(a) The phrase "absolute continuity" appears (as far as the reviewer can find) only in a parenthetic remark in what the authors call the Lebesgue-Nikodym theorem (known to most as the Radon-Nikodym theorem). The idea of absolute continuity is not exploited to the full, and with the one exception noted it is not identified by name.

(b) Necessary and sufficient conditions for mean convergence are mentioned only in an exercise (§5, Exercise 22) and even there the treatment leaves something to be desired. It is not pointed out that the conditions are necessary, and it is not pointed out that the three conditions can be reduced to two.

More important than these and other detailed objections is the fact that the authors still conceal the connection between measure theory and set theory. As in Chapters I–IV, measures are still pictured almost exclusively as linear functionals on spaces of continuous functions. Regardless of the merits of this approach to the subject, to raise a student on a diet of that and nothing else will probably retard his ability to communicate with the outside world. And, extol the virtues of Bourbaki as you will, there still is an outside world.

The reviewer feels very strongly that textbook authors, be they Bourbaki or mere mortals, should realize that their task is to supplement the literature, not to supplant it.

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Generators and relations for discrete groups. By H. S. M. Coxeter and W. O. J. Moser. Ergebnisse der Mathematik und Ihrer Grenzgebrete, New Series, no. 14. Berlin-Gottingen-Heidelberg, Springer, 1957. 8+155 pp. DM32.

It is refreshing to find a book that not only studies groups but also deals with many particular and interesting groups. The major theorems of group theory have substance only insofar as they apply to actual groups. The Mathematician with any feeling for groups will welcome this monograph and its rich display of groups of many kinds.

The monograph deals for the most part with finitely presented groups, that is groups G generated by a finite number of elements $R_1, R_2, \cdots, R_m$ subject to a finite number of defining relations $g_k(R_1, \cdots, R_m)=1$, $k=1, \cdots, s$. There are two main faces to the study of finitely presented groups. The obverse is the problem of studying the properties of a group defined by given relations. Among other things we wish to know if the group is finite and if so, what its order is. The reverse is the problem of finding a simple set of defining relations for a given group. Both these problems are studied in this monograph, and a variety of methods, mostly geometrical, are employed. Since the word problem for groups is unsolvable, we are relieved of the necessity of searching for an all embracing method and may enjoy the elegance of several diverse approaches.

A practical method of enumerating cosets is given. This method has the advantage that on completion it yields a permutation repre-