(Proc. Nat. Acad. Sci. U.S.A.); by Bott and Milnor (Bull. Amer. Math. Soc. vol. 64 (1958) pp. 87–89.; and by James (Proc. London Math. Soc.).]

Part III, *The cohomology theory of bundles*, contains an excellent exposition of obstruction theory, using bundles of coefficients. This theory is applied to the study of Stiefel-Whitney classes; however in view of the remarkable developments due to Thom and Wu, this approach to Stiefel-Whitney classes has become more or less obsolete. The next section studies the problem of whether or not a given n-manifold possesses a continuous field of quadratic forms of signature k. (I.e. does the tangent bundle split into the "Whitney sum" of a k dimensional vector space bundle and an $(n-k)$-dimensional vector space bundle?) Finally the Chern classes of a complex analytic manifold are considered.

The following is an illustration of the way in which material in Steenrod's book has led to important research. The problem of classifying sphere bundles over spheres occupies only six pages of the book. This led to work by James and Whitehead on the problem of classifying the resulting total spaces as to homotopy type. [See Proc. London Math. Soc. vol. 4 (1954) and vol. 5 (1955).] Hirzebruch used S^2-bundles over S^2 to show that the same differentiable manifold may possess several distinct complex structures. [Math. Ann. vol. 124 (1951).] The reviewer used S^3-bundles over S^4 to show that the same topological manifold may possess several distinct differentiable structures. [Ann. of Math. vol. 64 (1956).] Several authors have used S^3-bundles over S^4 to show that manifolds of the same homotopy type may be distinguished by their Pontrjagin classes. [R. Thom, Ann. Institut Fourier, Grenoble vol. 6 (1955–1956) p. 81; I. Tamura, J. Math. Soc. Japan vol. 9 (1957); and N. Shimada, Nagoya Math. J. vol. 12 (1957).]

In conclusion, this book remains a must for students of topology and geometry.

JOHN MILNOR

Differential equations: *Geometric theory* by S. Lefschetz. New York, Interscience Publishers, 1957. 10+364 pp. $9.50.

The present work is modestly referred to by the author as an extension of his previous Annals of Mathematics Study, *Lectures on differential equations*. Actually, the additional material included makes this a book which differs in character from its predecessor. The *Lectures on differential equations* was strictly a textbook for, say a first year graduate course in ordinary differential equations. The present volume, while it contains some introductory material (Chapters I–III), is primarily a treatise on the two aspects of differential equations theory which the Princeton Group in Differential Equations, founded by Mr. Lefschetz, have found most interesting. These topics are stability theory (Chapters IV–VIII) and two-dimensional systems (Chapters IX–XII).

The book thus serves a number of purposes: It can be used as a starting point for mathematical investigation in stability theory or two-dimensional systems. It may be useful to engineers and physicists who wish to inform themselves of the current state of knowledge in stability theory. It can be used as a text for a one-term course using Chapters I–III plus selected topics from the last four chapters. Finally, it is possible in a two-term graduate course in ordinary differential equations to spend the first term on linear equations and the second term on nonlinear equations. In this case it would be possible to use the book by Coddington and Levinson for the first term and the new book by Lefschetz for the second term.

As indicated, the book divides into three parts. The first three chapters and the appendix consist of introductory material. Chapter I and the appendix review the theory of functions of several variables as well as the topological and algebraic background material. Since the emphasis in the whole volume is on analytic systems the Weierstrass Preparation Theorem is introduced at this point. It later becomes an important tool in the study of singular points. The topological tools include metric spaces, differentiable manifolds, the index of a circuit, the Brouwer Fixed Point Theorem and the Poincaré Index Theorem. The review of linear algebra includes Jordan Canonical Forms and a discussion of logarithms of matrices.

In Chapter II the basic existence and uniqueness theorem is proved for the differential equation $dx/dt=f(x, t)$, where f is continuous and satisfies a Lipschitz condition. The solutions are then shown to be continuous functions of the initial conditions; they are differentiable (analytic) functions of the initial conditions if f is differentiable (analytic).

Chapter III is an introduction to linear differential equations. It has been previously pointed out that this book tends primarily towards nonlinear equations. This is perhaps best illustrated by the fact that the author motivates the study of linear differential equations by showing that linear differential equations may arise as the equation of first variation of non-linear equations. In line with this general outlook, linear differential equations are held to a minimum (20 pages). The author discusses the existence and uniqueness theory, the facts regarding the vector space of solutions, and the adjoint equation and its use in the solution of non homogeneous equations, linear systems with constant and periodic coefficients make up the rest of the chapter.

The most important part of this book is probably the section on stability, which is an excellent survey of the field. Not only is the choice of topics unusually good, but this section introduces the American reader to some recent work of Russian mathematicians, which has not been widely available. The section starts out, in Chapter IV, with the basic definitions of stability theory. The definitions are in line with those used by Malkin, Massera, and Antosiewicz. In Chapter V the basic stability theorem for the equation $dx/dt=Px+q$ is proved. Moreover, it is shown that a suitable process of successive approximations will tend towards a solution of this equation if the characteristic roots of P all have negative real parts and if q is suitably restricted. The rest of this chapter deals primarily with analytic systems and is built around the Liapunov Expansion Theorem. This Theorem describes a series solution of the nonlinear equation if the characteristic roots of P have negative real parts and are well behaved in the sense of Liapunov. The terms of this series are essentially bounded functions of t multiplied by certain exponentials which depend on the characteristic roots of P.

Chapter VI discusses the very important direct method of stability theory. This method consists of constructing something like a potential function near the trajectory under study. The author then develops some recent work by Persidski, Malkin, and Dychman. The theorem of Dychman can be used to study a two-dimensional nonlinear system where the coefficient matrix of the first-order terms has a zero root.

Chapter VII studies the stability properties of the system $dx/dt =P(t)x+g(x, t)$. In this case the stability properties of $dz/dt=P(t)z$ no longer determine the stability properties of the nonlinear system. The author uses a Theorem of Perron's to simplify the problem. This theorem of Perron's says roughly that the stability of the system is not changed if $P(t)$ is replaced by a similar triangular matrix. A number of sufficient conditions for the stability of the zero-solution of $dx/dt=P(t)x+g(x, t)$ are developed. The author proves that the various criteria for $dz/dt=P(t)z$ known as Persidski Conditions, Perron Condition, existence of a quadratic Liapunov Function are equivalent. They all imply stability for the non-linear system.

Chapter VIII deals with the stability theory of periodic systems. The basic theorem relating the characteristic exponents to asymptotic stability is proved. A special discussion of the autonomous system which is not covered by the general theorem follows. Quasi linear systems and complete families of periodic solutions are also included in this chapter.

The last few chapters are a very fine exposition of two-dimensional systems of differential equations. Chapter IX starts with an investigation of simple critical points. The index of simple critical points is computed. The chapter ends with a study of the Technique by which a differential equation can be extended from the Euclidean Plane to the Projective Plane, a method which has been neglected since Poincaré. Chapter X investigates general critical points. It is shown that for analytic systems the index of a critical point is equal to $1+1/2$ (number of elliptic section−number of hyperbolic section). The notion of the limit sets of a trajectory is taken up next. It is shown that the limit sets of a trajectory fall into four mutually exclusive categories. Critical points with a single zero characteristic root and structural stability are other highlights of this chapter.

Chapter XI discusses the equation $d^2x/dt^2+f(x)dx/dt+g(x)=e(t)$. We are concerned here with proving the existence and uniqueness of periodic solutions. The author exhibits a variety of techniques for accomplishing this goal, all taken from recent literature. Chapter XII studies the perturbation theory of second order differential equations. Sufficient conditions for the existence and uniqueness of periodic solutions of the perturbed systems are found. The stability of these periodic solutions is investigated. Other topics discussed in this chapter are the Stroboscopic method of Minorski, relaxation oscillation, and the stability zones of the Mathieu Equation.

Summarizing, this is a very interesting book containing a wealth of material. This work should be useful to a variety of mathematicians and physicists.

FELIX HAAS

Einführung in die transzendenten Zahlen. By. Th. Schneider. Berlin-Göttingen-Heidelberg, Springer, 1957. 7+150 pp. DM 21.60. Bound DM 24.80.

A complex number α is said to be algebraic or transcendental (over the rational numbers) according as it is or is not a root of an equation of the form $a_0x^n+a_1x^{n-1}+\cdots+a_n=0$, where $a_0, \cdots, a_n$ are rational numbers. The distinction between these two kinds of numbers was recognized at least as early as Euler (1744), who asserted that the logarithm to a rational base of a rational number must be either rational or transcendental. This was a bold conjecture indeed, since at that time no example of a transcendental number was known,