As can be seen from the above outline this book does not pretend to be a comprehensive treatment of the subject. However it gives a particularly lucid account of the topics it does treat. Until these lecture notes first appeared the geometric results in the last two chapters were only accessible to students in foreign texts, notably German.

EARL A. CODDINGTON

Commutative algebra, Vol. 1. By Oscar Zariski and Pierre Samuel. Van Nostrand, Princeton, 1958. 11+329 pp. $6.95.

In the last few decades the subject of commutative algebra has become increasingly important, both as a tool, especially in algebraic geometry, and as an area of research in its own right. Yet, surprisingly, until the appearance of the volume under review there has been no recent, adequate exposition of this far-reaching field. While van der Waerden's book has been of great significance in the training of many of the present day mathematicians, and while it does deal, in parts, with portions of commutative algebra, it is really designed for the very beginner and it does little more than scratch the surface. There are books dealing with ideal theory, for instance Krull's monograph or the volume by Northcott; however, one can fairly say that there was no single, up-to-date source to which the graduate student (or mature mathematician) could turn to learn commutative algebra. This situation is now changed, for this book by Zariski and Samuel more than adequately fills this vacuum. While it is difficult to predict its influence on the mathematical education of future mathematicians, this reviewer feels that it might very well play the role, in the time ahead, which van der Waerden did in the past.

Since there has been, and will be, great interest in this book, especially as a possible text for a graduate course we shall give a rather complete and detailed run-down of the contents, chapter by chapter.

Chapter 1 begins slowly with an introduction of the notions of group, ring and field and a development of some elementary, basic properties of these. It is interesting to note how the authors fret and worry about the question of embedding one ring in another. There follows a discussion of unique factorization domains. Polynomial rings are then introduced with very great care (in the beginning of the appropriate sections, informally as combinations of power products but in the body of the text, formally in terms of mappings) and the usual things about them are exhibited. Next to appear is the formation of quotient rings relative to a multiplicative system, and this concept is illustrated with several examples. The chapter closes with a discussion of vector spaces; of interest is the axiomatic treatment of dependence relations, which will subsequently lead to a unified treatment of the elementary properties of linear, algebraic and p-dependence.

The book really gets rolling in Chapter 2, which deals with field theory. The usual material about algebraic extensions is considered, e.g. the existence of splitting fields, algebraic elements, separability, etc. After defining normal extensions they take up the Galois theory and prove the fundamental theorem of the Galois theory. It is a pity that at this point the authors did not even choose to mention the connection of the theory to the problem of solvability by radicals. In fact, one complaint this reviewer would make is that the Galois theory is treated in an offhand and superficial manner; one of the most delightful, separate little packages of mathematics is passed over lightly in a cavalier way. The theorem of the primitive element is demonstrated in the highly transcendental method due to Kronecker; in consequence the more general theorem (which has found uses) where all but possibly one of the elements adjoined are separable does not drop out. Traces, norms and the discriminant now arise and their properties under extension and their relation to separability is considered. Up to this point in the chapter the reviewer feels that the authors have written with a heavy hand and that the extreme elegance and beauty of the elementary aspects of algebraic extensions fail to come through. Fortunately, at this point they turn their attention to the trascendental aspects of field theory, and this is handled superbly. Transcendence bases are defined and their existence, using Zorn's lemma, is shown. The transcendence degree is now due to appear—and it does, right on schedule. Its properties are developed and used, for instance, to prove the result of F. K. Schmidt that if k is perfect then $k(x_1, \cdots, x_n)$ can be separably generated over k. The sections of this chapter which appealed most to the reviewer were those dealing with linear disjointness and its relation to separability. These parts are developed with loving care and thoroughness. The chapter ends on a rather unusual note—a discussion of the derivations of fields. A good part of this material was new to the reviewer and he found it all fascinating.

Chapter 3 handles ideals and modules. In contrast to the preceding chapter, where every paragraph was packed with content, the exposition here rambles along at a quiet, leisurely pace. The old familiar landmarks—radicals of ideals, prime ideals, primary ideals, chain conditions—come into view. We even catch sight of the Chinese

Remainder Theorem expressed in terms of ideals. The authors now consider a topic which has become just as familiar a part of the mathematical landscape—tensor products, and treat this very important construction with great care. The chapter concludes with a discussion of the free join of integral domains relative to a given field. In order to examine this they introduce a generalization of linear disjointness—quasi-linear disjointness—and using this idea they characterize the situation when any two free joins of two integral domains relative to a given field, k, are equivalent. As corollaries of this characterization they present us with some familiar results, such as, for instance, the theorem that if K and K' are extension fields of k and if one of these is separable over k then $K \otimes K'$ is free of nilpotent elements.

Chapter 4 is the first one which might be said to treat pure commutative algebra, for it concerns itself with Noetherian rings. The theory of Noetherian rings has, of course, many high points, and they start the chapter with one of these, the Hilbert Basis Theorem (giving two proofs). As an aside they present the theory of commutative rings with descending chain conditions and derive their structure in terms of primary rings. The very important decomposition of an ideal as the intersection of primary ideals is then given, and the uniqueness of the various associated prime ideals is presented. As an application the result of Krull which gives the conditions that the intersection of the powers of an ideal be (0) is given. In a few side remarks they point out that they could use the powers of an ideal as a system of neighborhoods of (0) to introduce a topology on R, laying the ground work for the parts on local rings which will appear later. The previously defined concept of quotient ring relative to a multiplicative system is is extended in order to go into a detailed study of the relation of the ideal structure of a ring R with that of its quotient ring R_M relative to a multiplicative system M. Among other things it is shown that R Noetherian implies that R_M also is. A 1–1 correspondence is set up between the set of prime ideals of R which are disjoint from M with the set of prime ideals of R_M. If $\mathfrak{A}$ is an ideal of R, and $\mathfrak{A}^e$ the ideal of R_M generated by $\mathfrak{A}$ it is proved that if M does not intersect $\mathfrak{A}$ then the rings $R_M/\mathfrak{A}^e$ and $(R/\mathfrak{A})_M$ where $M' = (M + \mathfrak{A})/\mathfrak{A}$ are isomorphic. This leads to a detailed discussion of the situation where M is the complement of a prime ideal. The concept of length of an ideal now arises and is studied. The principal ideal theorem which states that the isolated prime ideals of a proper principal ideal are minimal prime ideals is demonstrated. Principal ideal rings and domains and irreducible ideals are scrutinized in turn. The chapter ends with an appendix on Noetherian modules.

Chapter 5, the last chapter, is a study of Dedekind rings. Integral dependence in rings is introduced and the transitivity thereof is established; from this the authors go on to integrally closed domains. The following result (Cohen-Seidenberg) relating the prime ideals of a ring with those of an overring which is integral over it appears; let A be an integrally closed domain and $A' \supset A$, integral over A and with the same unit element as A, and such that no element of A is a zero divisor in A'; if p and q are prime ideals of A, $p \subset q$, and if p' is a prime ideal of A' lying over p then there is a prime ideal $q' \supset p'$ of A' lying over q. Finite integral domains, that is, those of the form $k[x_1, \cdots, x_n]$ for k a field are considered and the normalization theorem proved for them. A Dedekind ring is defined as an integral domain in which every ideal is the product of prime ideals; this decomposition is shown to be unique. En route to this fact the idea of an invertible ideal arises and another characterization of Dedekind rings, namely, that every ideal is invertible, is given. A somewhat complicated proof is given (a proof using valuation theory is promised to be given in the second volume) of the theorem: if R is a Dedekind ring, K its quotient field and L a finite algebraic extension of K then the integral closure, R', of R in L is a Dedekind ring. Given a prime ideal p of R consider the factorization of $R'p$ in R'; this leads to the reduced ramification index, e_i, of prime ideals $\mathcal{P}_i$ in R' over p. After introducing the relative degree f_i of $\mathcal{P}_i$ over p, it is proved that $\sum e_i f_i \leqq [L:K]$, and a condition for equality is given. As a corollary equality is proved for separable extensions. Turning to the Gaussian integers they use these ideas to prove the Fermat two square theorem. The decomposition group, inertia group and ramification group for finite, separable normal extensions are introduced. For R an integrally closed Dedekind ring, K its quotient field, and K' a finite, separable, algebraic extension of K, and $R' \subset K'$ integral over R, the different ideal is defined. In the case that R' is the integral closure of R, denoting the different by $D_{R'/R}$, the familiar characteristics of $D_{R'/R}$ are exhibited and ramification for prime ideals in R' is shown to be equivalent to dividing $D_{R'/R}$, leading to the finiteness of the number of prime ideals in R' which are ramified. The discriminant of R' over R appears in terms of the norm of $D_{R'/R}$. If R is a Dedekind ring with quotient field K, and K', K'' finite algebraic separable extensions of K such that $K' \subset K''$ then $D_{R''/R} = D_{R''/R'}(R''D_{R'/R})$, and an analogous result holds for the discriminant. Turning to the special case of quadratic extensions of the rationals, $K = Q(m^{1/2})$, m a square free integer, they work out the ramified, inertial and decomposed primes. Considering cyclotomic fields they derive the quadratic reciprocity law. The book closes with a theorem of Kummer.

The book should easily adapt itself as a text for a year course, early in the training of a graduate student. The reviewer feels that the average graduate student will find the book difficult to read but that his efforts will be well compensated. The book has enough rough edges so that the student will see the work and effort that go into a piece of mathematical work, something he will seldom appreciate in the very highly polished pieces of art sometimes presented to him when he is not quite ready to digest them or to benefit from them. The book is not written as a "text" book, so is devoid of problems. The book also suffers, especially in the earlier parts, from a lack of discussion of examples. Both these faults can easily be remedied by the introduction of supplementary material by the instructor. The instructor will, in many cases, feel a need to amplify the portions of the book dealing with Galois theory. Since the authors left valuation theory for a later volume one might have to supply material on this topic if it is wanted in the course.

Nowadays the fashion in the teaching of our graduate students seems to have become their introduction to highly formal gadgetry at a very early stage of the student's development. The greatest merit of this book is that it is down to earth and genuine. Its acceptance as a standard text should be a force in checking the drift to higher and higher formal structures and abstraction upon abstraction. The authors set out to fill a void in the mathematical literature. They have done this admirably.

I. N. HERSTEIN

Ordinary difference-differential equations. By Edmond Pinney. University of California Press, Berkeley 4, California, 1958. 12+262 pp. $5.00.

This work represents the only exposition, so far as the reviewer is aware, of the recent development of difference-differential equations stimulated by modern electronic control mechanisms. The author presents his own development of the subject, which in some places includes previously unpublished results. He gives a good list of references to the many recent papers published on the subject. He has not written a research treatise, in that he does not attempt even to state all the results achieved in these papers, many of which are not included in his own development. This book could serve as a text