

## ON OPEN MAPPINGS IN BANACH ALGEBRAS, II

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This paper contains an elementary way to look at the results of [1] and [2]. We prove Theorems 4 and 5 of [2].

Let  $\mathfrak{A}$  be a Banach algebra,  $f$  a holomorphic function on an open set  $U$  in the plane,  $a \in \mathfrak{A}$ ,  $\sigma(a) \subset U$ , and  $f(\sigma(a)) \subset f(V)$  where  $V$  is an open set on which  $f$  is 1-1. Then a neighborhood of  $f(a)$  consists entirely of points of the form  $f(b)$  for some  $b \in \mathfrak{A}$ ; further, if  $\sigma(a_1) \subset V$  and  $f(a) = f(a_1)$ , then  $a$  and  $a_1$  commute.

To prove this, let  $\tilde{f}$  be the restriction of  $f$  to  $V$ ,  $\tilde{f}^\vee$  the inverse function to  $\tilde{f}$ .  $\tilde{f}^\vee$  is defined on an open set  $W$  of the plane, with  $f(\sigma(a)) \subset W$ . There exists a neighborhood of  $f(a)$  all of whose elements have spectrum contained in  $W$  (see e.g., [2, Lemma 2]). For these elements  $c$ , set  $b = \tilde{f}^\vee(c)$ . Then  $f(b) = \tilde{f}(b) = \tilde{f}(\tilde{f}^\vee(c)) = c$ .

The second assertion comes from noticing that  $a$  commutes with  $g(a)$  for any  $g$  holomorphic on  $U$ ; in particular, with  $\tilde{f}^\vee(f(a)) = \tilde{f}^\vee(f(a_1)) = a_1$ .

### BIBLIOGRAPHY

1. E. Hille, *On roots and logarithms of elements of a complex Banach algebra*, Math. Ann. vol. 136 (1958) pp. 46-57.
2. C. McCarthy, *On open mappings in Banach algebras*, to appear in the Journal of Mathematics and Mechanics.

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