theorem and the Stone-Čech compactification. There is a table (stretching over six pages) that summarizes the main results. Chapter V, *Convex sets and weak topologies*. The main topics are: separation theorems for convex sets, the Tychonoff-Alaoglu theorem, Eberlein's theorem on sequential compactness, the Krein-Milman theorem, and the Schauder fixed point theorem. For locally convex spaces, only what might be called the "classical" theory is given—pre-Bourbaki and pre-Grothendieck.

Chapter VI, *Operators and their adjoints*. Main topics: completely continuous operators, and the Riesz-Thorin convexity theorem. There is a tabular presentation of the principal representation theorems for operators from and to the standard spaces. Chapter VII, *General spectral theory*. The discussion begins with finite-dimensional spaces, includes the Riesz theory of completely continuous operators, and includes also an introduction to perturbation theory. Chapter VIII, *Applications*. The applications concern semigroups (the Hille-Yosida theorem), and ergodic theory (mean and individual).

The terminology and the notation are almost always standard and easy to assimilate. The exposition is never watery. Things proceed at a good clip; the definitions and the proofs are concise and neatly formulated. A tremendous enterprise such as the authors have undertaken is more likely to fail than to succeed, and existence itself is more than half the proof of success. The authors deserve thanks for their labors and congratulations on their achievement.

PAUL R. HALMOS

Introduction to logic. By Patrick Suppes. Princeton and New York, Van Nostrand, 1957. 18+312 pp. $5.50.

One can distinguish at least three attitudes towards the increasingly important role of logic in the undergraduate mathematics curriculum; the *reactionary* attitude which denies it any place; the *moderate* attitude which regards it as a "luxury" subject, to be made available to those advanced students who are especially interested; and the *progressive* attitude which regards it as one of the earliest and most basic skills which a major should learn.

Rosenbloom's *Elements* is excellent for the purposes of the moderates; but until the appearance of Suppes' book the progressives had the alternatives of teaching from notes or bowdlerizing one of the existing texts. The better of those texts shared the feature that they became embroiled in logical questions for their own sake rather than as a tool of mathematics; some of them (notably Fitch's and Rosser's) fell on this account awkwardly between being an undergraduate text and being a research contribution. Others (like Tarski's) were too informal to contribute much to the student's nascent feeling for rigour, while yet others (like Quine's) emphasized rigour so much that from the student's standpoint (and quite likely sometimes from the teacher's) they had no connection with any other mathematical work.

Because of these obvious inadequacies in all previous texts for the purpose (namely basic logical training of undergraduate science majors) which the progressives have in mind, it is ridiculous that they should use for these purposes any other text than Suppes.

The book is divided into two parts, the first on the functional calculus and the second on elementary informal set-theory. It is possible to teach the first part in one semester, and most of the second in another.

Chapters I and II deal with those principles of deduction which are grounded in the propositional calculus. The emphasis throughout being on logic as a tool of proof rather than an independent discipline, the propositional calculus itself is not presented. The deductive methods used are a combination of truth-tables and natural deduction. Methods of testing for truth-functional consistency are provided, and the student is introduced early to the nature of an *interpretation*. Though this is not necessary at the truth-functional level, it is a good idea pedagogically since it serves to make painless the introduction in Chapter IV of the important idea of proving consistency by a model.

Chapter III introduces quantifiers and discusses the translation of sentences (mostly nonmathematical) into the notation of symbolic logic. Chapter IV presents rules for the first-order functional calculus; a smooth-running natural deduction technique is developed. In an endeavour (which springs naturally from the underlying philosophy of the book) to make the formal proofs look as much as possible like ordinary informal mathematical proofs, an analogue to the Hilbert epsilon-symbol is introduced. It corresponds rather closely to the symbols x_0, n_0, etc. used for existential instantiation in informal proofs. Such little pedagogical tricks as this, while in no way affecting the logico-mathematical content of the book, help make it much more palatable and less distracting to the student. Another instance of the same tendency is seen in the inclusion of redundant rules (e.g. one permitting the replacement of (x) by $-(\exists x)-$ and $(\exists x)$ by $-(x)-$, and another permitting the replacement of any condition by any equivalent condition); the desire for economy or elegance which prevents other texts from following this course is really quite out of place in a book for the nonspecialist. As a last illustration: constants and operation-symbols are not required to be replaced by predicates, as they too often are by other writers.

Chapter IV also contains a noteworthy section on proving consistency (or what comes to the same thing, invalidity or independence) by means of a model.

The presentation of the formal rules of deduction is completed in Chapter V, which contains the rules for identity and one or two other supplements. It is noteworthy that the notion of a *theorem of logic* does not appear before this point (i.e. not before there is a good reason to introduce it, namely the abbreviation of proofs). This is in refreshing contrast to the procedure of many authors of undergraduate logic texts, who conceive their purpose to be primarily the proving of theorems of logic, and who consequently introduce the notion, with little motivation and to the confusion of the student, on page 1 of their treatises.

Chapter VI is called *Postscript on use and mention*, which adequately describes its purpose. In Chapter VII, *Transition to informal proofs*, the author establishes the relevance of the techniques previously set up to the kind of mathematics which is likely to be familiar to the student. The two main lessons of this key chapter are (1) that no alleged proof can be accepted unless it can *trivially* be made into a formal proof and (2) that for all that, full formalization is neither necessary nor desirable. These points are illustrated by a detailed working-out of the first properties of ordered fields; the text assumes a more "mathematical" appearance from here on, partly because of the omission of initial universal quantifiers. A really helpful section on fallacies in informal proofs completes the chapter. (Most texts either offer a useless and archaic treatment, or none.)

Chapter VIII on Definition is the most original in the book. It contains detailed rules for introducing defined symbols into first-order systems. The author's philosophy is intermediate between the two classical ones; like Lesniewski, and unlike Russell and Bourbaki, Suppes regards definitions as new axioms rather than as mere abbreviations; on the other hand, like Russell and Bourbaki, and unlike Lesniewski, he requires that they be conservative and eliminable. This position is identical with that tacitly adopted in working mathematics; the chapter will be very useful to students who are puzzled by such matters as division by zero (to which a six-page illustrative discussion is devoted). The theory is sufficiently flexible to include partial definitions (which are much commoner than most logicians care to admit). A section on the use of Padoa's method to prove independence of primitive ideas completes the chapter.

Part II (except for the last chapter) is concerned with the beginnings of (intuitive) set-theory. Again, emphasis is on elementary applications of set-theory to mathematics rather than on set-theory as an independent discipline. (A sequel on the latter topic is planned.) Sets of sets are scarcely within the scope of the book. An informal style is adopted, as is natural after Chapter VII.

Chapter IX presents the basic laws of the (finite) algebra of sets. These laws are deduced from the definitions of the operations, or established by Venn diagrams: the axiomatic treatment is relegated to exercises. Chapter X discusses the basic notions of the theory of relations (regarded as classes of ordered pairs). The central topic is classification of relations (various kinds of orderings, equivalence relations); such less familiar notions as relative product and converse also receive adequate attention. Chapter XI, on functions, should succeed in scotching once for all the student's traditional confusion over this weasel notion. Final clarification is attained in the last $3\frac{1}{2}$ pages on Church's lambda-notation; rather than suggest omission of this section, as Suppes does, I would make it required reading for any teacher of elementary calculus who has any lingering doubts as to just what a function is.

Chapter XII is the longest (59 pages) and most controversial in the book. It deals with axiomatization, taking the position that mathematical disciplines are to be axiomatized within set-theory (which is itself not axiomatized in the present treatment). An axiom-system is thus a kind of definition ("$\langle A, \circ \rangle$ is called a group if . . . "). A good deal of space is taken up with the question of isomorphism and representation and unique-representation theorems, using the Cayley theorem as an example. The underlying philosophy of *hypothetical* mathematics is not unlike the theory of "theories" in Bourbaki's *Ensembles*. (The fact that *absolute* mathematics (arithmetic and analysis) admits also of a reduction to set-theory is hardly mentioned: this is another concession to the student's needs that few logicians have cared to make.) The chapter concludes with a detailed working-out (partly in exercises) of two axiomatic systems for applied disciplines, one for probability and one for Newtonian mechanics. Like the chapter on definition, this one is a "first" in doing something which should have been done in every nonspecialized book on logic (if by "logic" we mean a study designed to give the student an initial apprehension of the most basic mathematical procedures, which he will meet over and over again throughout his subsequent studies); and like the chapter on definition, it is altogether (apart from the section on mechanics) in line with current (tacit) practice. Just because this practice *is* tacit rather than explicit, Suppes' book will equip the student at the outset of his training with perspectives which normally come to him only after the attainment of considerable maturity.

I list the only criticisms I can think of. Some of the exercises may be considered too hard (pp. 17, 207, 305) and some of the applications too specialized (pp. 265–271) or too controversial (pp. 291–305) for inclusion in an elementary text. One of the exercises (ex. 1 on p. 194) is likely to lead to confusion without some fairly detailed discussion on the part of the instructor. (E.g. does "All predicted responses were reinforced" mean, "It was predicted that all responses would be reinforced," or, "All those responses which were (correctly) predicted were (predicted and observed to be) reinforced"?) Most instructors will find it desirable to add some mathematical illustrations to the almost exclusively verbal ones in Chapters I–VI. One or two choices of subject-matter are perhaps slightly questionable; for example I would rather see an elementary treatment of homomorphism around p. 220 than the curious discussion of the computation of the inverse of a (real) function which occupies pp. 235–240, and I would rather see Chapter VI on use and mention curtailed and embodied in the chapter on functions (which is what most instructors will do with it anyway). Finally the quantificational rules of Chapter IV can be somewhat simplified with no loss in rigour (e.g. along the lines of Fitch's *Symbolic logic*, §§ 21.12–13 and 22.8–9).

But all these are very minor quibbles reflecting as much as anything biographical idiosyncrasies and accidental encounters with students. Try as I will, I cannot find anything serious to compain about. The book comes as near to a perfect fulfillment of its function in the rough-and-tumble of the classroom as any you are likely to find. Clearly it is destined to become a classic and not be soon replaced. I can only hope that it will stimulate educators to try the effect of an early rigorous logical training, perhaps compulsory, on science majors generally. It was a thankless task indeed to make this experiment before the publication of Suppes' book; now it has become a challenge and an adventure.

NOTE. The first printing (1957) was marred by a large number of printer's errors. However of those which caught the attention of this reviewer, only two remain in the second printing (1958). P. 167, l. 7, replace "first" by "second"; p. 172, l. 7 from bottom, replace "independent of" by "dependent on."

JOHN MYHILL

Asymptotic methods in analysis. By N. G. de Bruijn. Amsterdam, North-Holland; Groningen, Noordhoff; New York, Interscience, 1958. 12+200 pp. $5.75.

This book is for you if you are interested in answering questions like the following: What is a good approximate formula for x if