nection with non-integral n," thus (since Y_n is not introduced) leaving one helpless in problems about annuli. Again, the authors' treatment of the calculus of variations is in the metaphysical style of the seventeenth century, with "variations" that are sometimes zero and sometimes not. This is inexcusable if the idea is to prove anything, but perfectly all right if the idea is simply to furnish a mnemonic for Euler's equation.

The authors say in their preface, "The degree of rigor to which we have aspired is that customary in careful scientific demonstrations, not the lofty heights accessible to the pure mathematician." That is, this is a book about pidgin mathematics; as such, it will not appeal to any standard mathematician who may be looking for a text in applications of mathematics. It may, indeed, give him qualms about the validity of "careful scientific demonstrations." I need say little more, since practitioners of pidgin mathematics are unlikely to read this *Bulletin*. The first edition (reviewed in Bull. Amer. Math. Soc. vol. 57 (1945) pp. 508–509) was enormously successful. The second edition differs from the first chiefly by the addition of a section on Fourier and Laplace transforms. Parts of the book are primarily physical (thermodynamics, mechanics of molecules, quantum mechanics, statistical mechanics); some are handbook-style collections of facts (vectors, tensors, coordinate systems, matrices, numerical methods, and the parts of group theory that are too advanced for elementary texts and too special for advanced ones); some consider mathematical tools (differential equations, special functions, calculus of variations, integral equations). The physical parts seem lucidly written and can even be read by mathematicians who want to acquire a smattering of physics to impress their friends. The rest seems adequate within the setting for which it was designed, although even so some physicists have not found it altogether satisfactory as a text; perhaps this corresponds to the fact that (for example) a text written in Melanesian Pidgin would cause difficulties for a reader of Australian Pidgin.

R. P. Boas, Jr.

Computability and unsolvability. By Martin Davis. New York, McGraw-Hill, 1958. 25+210 pp. $7.50.

This book gives an expository account of the theory of recursive functions and some of its applications to logic and mathematics. It is well written and can be recommended to anyone interested in this field. No specific knowledge of other parts of mathematics is presupposed. Though there are no exercises, the book is suitable for use as a textbook.

Let us restrict the meaning of the words "function of n arguments" to that of a mapping of the collection of all ordered n-tuples of non-negative integers into the collection of all non-negative integers. A function is *effectively calculable*, if there exists an algorithm (i.e., systematic procedure) by which one can compute the value of the function for any given ordered n-tuple of non-negative integers in a finite number of steps. The concept of an effectively calculable function is informal, since its definition involves the unexplained notion of an algorithm, but it has a strong intuitive appeal. A certain denumerable family of functions is specified as the family of all *recursive* functions. For this family there exists a precise, formal definition which meets the modern standards of rigor. Recursive functions derive their importance from the fact that there is overwhelming evidence that *a function is effectively calculable if and only if it is recursive* (Church's thesis). Various formalizations of the intuitive concept of an effectively calculable function have been proposed in the nineteen thirties (Church, Gödel, Kleene, Post, Turing) and their equivalence has been established.

Martin Davis bases his exposition on the notion of a Turing machine. Whether this is the best way to present the subject is a matter of taste, but it does make his book valuable to persons working in the field of computing machines. It is to be hoped that some of them will now investigate which recursive functions are of *practical* importance to the design of computers.

The book is divided into three parts and eleven chapters. A brief outline follows. There is an introduction in which the nature of decision problems is explained together with the notations to be used throughout the book. Chapter 1 opens with the definition of a Turing machine. A function is called *computable*, if its values can be computed by such a machine. For each of several functions, e.g., $x+1$, $x+y$ and $(x+1)(y+1)$, a Turing machine which can compute it is described in great detail. There follows a discussion of relative computability. In Chapter 2 general methods are given which yield the computability of all members of a large family of functions. Recursive functions are introduced in a purely number-theoretical fashion in Chapter 3, together with primitive and partial recursive functions, recursive sets and predicates. It is shown in Chapter 4 that the technical notion of a recursive function is equivalent to that of a computable function. The existence of a universal Turing machine is established using the familiar arithmetization technique. Chapter 5 deals with unsolvable decision problems and recursively enumerable sets. It is proved that there exists an infinite recursively enumerable set (i.e., a set of non-negative integers which is the range of some one-to-one recursive function) whose characteristic function is not recursive. In fact, two such sets are exhibited, namely a creative set and a simple set using variants of Post's well-known proofs. This chapter concludes the first part of the book.

Part II is concerned with three applications of the theory so far developed. Chapter 6 treats the unsolvability of various combinatorial problems, in particular the word problem for semigroups. The unsolvability of the word problem for groups (shown by Novikoff and, independently, Boone) is stated, but not proved. Chapter 7 is devoted to Hilbert's tenth problem, i.e., the problem whether there exists an algorithm for deciding whether any polynomial equation of the form

$$P(x_1, \cdots, x_n) = 0,$$

where P has integers as coefficients, has a solution in integers. This problem is still unsolved. Here the author also presents his own results on diophantine predicates. This part closes with Chapter 8 which deals with mathematical logic; it contains a proof of Gödel's incompleteness theorem and establishes the existence of a first-order logic with an unsolvable decision problem.

Part III continues the development started in Part I. It is concerned with the Kleene hierarchy, computable functionals, the recursion theorems, and degrees of unsolvability. Post's problem is stated and its solution (by Friedberg and, independently, Mucnik) mentioned, but not presented. The last chapter ends with a treatment of recursive ordinals and a few remarks about extensions of the Kleene hierarchy. There is an appendix in which some number-theoretical theorems needed in the text are proved in detail, namely, the prime factorization theorem and the Chinese remainder theorem. Finally, there is a four-page bibliography and an index.

J. C. E. DEKKER

P. R. Halmos, *Lectures on ergodic theory*. Tokyo. The Mathematical Society of Japan, 1956. 7+99 pp. $2.00.

This book is the first work on ergodic theory to appear in book form since E. Hopf's Ergodentheorie appeared in 1937. Its contents are based on a course of lectures given by the author at the University of Chicago in 1955. The first of these facts makes the book very welcome. More so since the book is written in the pleasant, relaxed