

Since injective sheaves are necessarily flabby, it is seen that the cohomology theory defined in this book agrees with that defined by Grothendieck.

As was said before, this book was clearly and carefully written. As is usually the case with a reference book, however, it reads like one. For one who knows in which direction all this material is heading, the clarity of presentation makes the reading not unpleasant. For students, I think that this book would best be used in conjunction with a seminar, a course, or dextrodine. I fervently hope that volume II of this work will appear soon. If it is as well written as this volume, I think that the author will have done us all a great service.

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Appendice—Résolutions simpliciales standard.

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Éléments de mathématiques. By N. Bourbaki. Book II, *Algèbre*. Chapter VIII, *Modules et anneaux semi-simples*. Actualités Scientifiques et Industrielles, no. 1261, Paris, Hermann, 1958.

In the volume under review the polycephalic author has written a beautifully clear exposition of that part of the structure theory of associative rings that he feels is of the greatest interest in itself and also of widest applicability in other branches of mathematics. Although the main stress is on rings with minimum condition, full advantage has been taken, as will be seen below, of the recent work on the structure theory of general rings to simplify and clarify the more classical results.

In accordance with the now almost universally accepted conventions, ring always means ring with unit and every module is unitary.

The first five paragraphs deal mainly with the theory of semi-simple (completely reducible) modules: If A is any ring and M a left A -module, the ring $C = \text{Hom}_A(M, M)$ is called the "commutant" of M and $B = \text{Hom}_C(M, M)$ is called the "bicommutant" of M . These notions are studied in detail, special attention, of course, being paid to those cases in which $A = B$. When M is a direct sum of modules all isomorphic to a fixed module M_0 (M is "isotypic"), the problem of describing C and B in terms of the commutant and bicommutant of M_0 is especially well handled without the use of bases by means of the functors \otimes and Hom . There follows a brief discussion of the ascending and descending chain conditions for rings and modules (or in the author's terminology: noetherian and artinian rings and modules). Next, semisimple and simple (completely reducible and irreducible) modules are introduced. The Chevalley-Jacobson density theorem is proved in the following neat and general form: Let M be a semisimple A -module and b an element of the bicommutant of M . For every x_1, \dots, x_n in M there is an a in A such that $ax_i = bx_i$, $i = 1, \dots, n$. By use of the earlier machinery, the standard theorems on the commutant and bicommutant of semisimple modules follow readily. A ring A is called semisimple (simple) if it is semisimple as a left module over itself (semisimple and has only 0 and itself as two-sided ideals) and the two main Wedderburn theorems then are quickly deduced. The section closes with a study of the notion of "height" and "index" of a simple subring of a simple ring.

Next, the notion of the (Jacobson) radical is treated: Let M be a left A -module; $\text{rad } M$, the radical of M is defined to be the intersection of all the proper maximal submodules of M , $\text{rad } A$ is then the radical of A thought of as a left module over itself. The basic properties of $\text{rad } M$ are proved, special attention, of course, being paid to the case where A is a ring with descending chain condition. Here then come the usual characterizations of semisimple (simple) rings as rings A with descending chain condition and $\text{rad } A = 0$ ($\text{rad } A = 0$ and no proper two-sided ideals), as well as Hopkins' theorem that a ring satisfying the descending chain condition also satisfies the ascending chain condition.

The general theory is then applied to the radical of a tensor product. The results here push somewhat beyond what was previously to be found in the literature. For example, it is shown that $(\text{rad}(A \otimes_K N)) \cap N = \text{rad } N$ where K is a field, A is a locally finite K -algebra and N is a finitely generated B -module for any K -algebra B . The struc-

ture of the tensor product of two semisimple modules is studied, special attention being devoted to finite dimensional modules. There follows the general notion of separability: Let K be a field, A a K -algebra, and M an A -module. If for every field extension E of K , $A \otimes E$ or $M \otimes E$ have zero radical they are called separable, and all the standard consequences are quickly given. This part of the book closes with a treatment of the composite of two fields and the decomposition of commuting sets of endomorphisms into semisimple and nilpotent parts.

The next paragraph deals rather succinctly with the Skolem-Noether theorem and the centralizers of simple subalgebras of central simple algebras. Splitting fields are defined and the existence of separable splitting fields as well as of the Brauer group is proved.

The final two sections deal with norms and traces and the basic notions of representation theory. The treatment of the first topic is more complete and substantial than any other that appears in the literature, and should, therefore, be very useful.

The text proper terminates with an appendix showing how the notions of radical and simple module may be extended to rings without units by means of Segal's notion of "regular" ideals.

As is usual in the works of Bourbaki, the exercises supplement the text very greatly. For example, the theory of primitive rings, the Galois theory for simple rings with descending chain condition, and the problem of raising idempotents modulo a nil-radical, are all treated there. Indeed, although the author claims in his historical note to have kept only the theory of the radical and the density theorem from the structure theory of rings without chain condition, this is only true as far as the main text goes. The exercises seem to include at least parts of most of the more important papers in ring theory of the past fifteen years. The expert will find it amusing to attach bibliographical references to individual problems and the less experienced reader will gain both valuable information and techniques through careful study and attempts at solution of the exercises.

It is no doubt partly due to the subject matter, but this volume seems, at least to this reviewer, to be less arid than the earlier ones in the algebra series. Neither is it too stuffed with results; the author has very nicely balanced his avowed aim of complete rigor and abstraction with enough concrete examples and interesting theorems to make the reading of this book satisfying and pleasant.

I should like to carp a little at the wholesale introduction of new terminology: "artinian" is admittedly briefer than "with descending chain condition" but hardly more informative. "Isotypic" in place of

“homogeneous,” “simple” in place of “irreducible,” “countermodule” for the same group as a module over the commutant, as well as several others seem to me to violate one of Bourbaki’s stated policies: never change or introduce new terminology without very serious reasons.

A slightly more serious drawback of the book is that not enough attention has been paid to algebras over arbitrary commutative rings, attention being restricted almost exclusively to algebras over fields. In view of recent developments, treatment of the more general case would have been desirable.

However, in conclusion I can do no better than to agree with Artin’s statement, in his review of the first seven chapters of Bourbaki’s *Algèbre* in the 1953 volume of this Bulletin, that a complete success has been achieved in this part of the work. Indeed, in the volume under review I do not even feel that the presentation is “mercilessly abstract” and without doubt “the reader . . . will be richly rewarded for his efforts by deeper insights and fuller understanding.”

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Univalent functions and conformal mapping. By James A. Jenkins.

Ergebnisse der Mathematik und ihrer Grenzgebiete, new ser., no. 18, Berlin-Göttingen-Heidelberg, Springer, 1958. 8+169 pp. DM 34.

Let S denote the family of functions $f(z)$, regular and univalent in $|z| < 1$, $f(0) = 0$, $f'(0) = 1$, $f(z) = z + \sum_{n \geq 2} A_n z^n$, and let Σ denote the family of meromorphic functions, univalent in $|z| > 1$ and with Laurent expansion in a neighborhood of the point at infinity $f(z) = z + a_0 + \sum_{n \geq 1} a_n z^{-n}$. The present monograph is concerned with the study of functions belonging to S , Σ , or related families. The author’s own contribution to this theory has been quite substantial and most, if not all results quoted in the book, without a bibliographical reference, are the author’s own. In view of his modest claims in the preface, the reader will not expect an exhaustive treatment of the theory of univalent functions, something obviously impossible in 160 pages of text.

As stated by its author, the monograph centers around the GCT (General Coefficient Theorem). But the first, introductory chapter contains a rather complete survey of results and methods in the theory of univalent functions. The non-elementary methods are grouped into four broad categories and are sketched in sufficient detail to give the nonspecialist a good grasp of the fundamental ideas in-