"homogeneous," "simple" in place of "irreducible," "countermodule" for the same group as a module over the commutant, as well as several others seem to me to violate one of Bourbaki's stated policies: never change or introduce new terminology without very serious reasons.

A slightly more serious drawback of the book is that not enough attention has been paid to algebras over arbitrary commutative rings, attention being restricted almost exclusively to algebras over fields. In view of recent developments, treatment of the more general case would have been desirable.

However, in conclusion I can do no better than to agree with Artin's statement, in his review of the first seven chapters of Bourbaki's *Algèbre* in the 1953 volume of this Bulletin, that a complete success has been achieved in this part of the work. Indeed, in the volume under review I do not even feel that the presentation is "mercilessly abstract" and without doubt "the reader . . . will be richly rewarded for his efforts by deeper insights and fuller understanding."

ALEX ROSENBERG

Univalent functions and conformal mapping. By James A. Jenkins. Ergebnisse der Mathematik und ihrer Grenzgebiete, new ser., no. 18, Berlin-Göttingen-Heidelberg, Springer, 1958. 8+169 pp. DM 34.

Let S denote the family of functions $f(z)$, regular and univalent in $|z|<1$, $f(0)=0$, $f'(0)=1$, $f(z)=z+\sum_{n\geq 2} A_n z^n$, and let Σ denote the family of meromorphic functions, univalent in $|z|>1$ and with Laurent expansion in a neighborhood of the point at infinity $f(z)=z+a_0+\sum_{n\geq 1} a_n z^{-n}$. The present monograph is concerned with the study of functions belonging to S, Σ, or related families. The author's own contribution to this theory has been quite substantial and most, if not all results quoted in the book, without a bibliographical reference, are the author's own. In view of his modest claims in the preface, the reader will not expect an exhaustive treatment of the theory of univalent functions, something obviously impossible in 160 pages of text.

As stated by its author, the monograph centers around the GCT (General Coefficient Theorem). But the first, introductory chapter contains a rather complete survey of results and methods in the theory of univalent functions. The non-elementary methods are grouped into four broad categories and are sketched in sufficient detail to give the nonspecialist a good grasp of the fundamental ideas involved; generous bibliographical references are also very helpful. Besides the earlier, now largely classical, papers (elementary approach, only results stated, no indication of methods given) of Koebe, Hurwitz, Plemelj, Gronwall, Bierberbach, Faber, Löwner, Szegö, Littlewood and Szegö, the author mentions the work of Prawitz (generalized area principle); Löwner, Golusin, Peschl, Fekete, Szegö, Schaeffer and Spencer, Robinson, Basilevitch (parametric method); Grötzsch, Ahlfors, Rengel, Beurling, Golusin, Teichmüller, Ahlfors and Beurling (extremal metric); Grunsky, Bermant, Golusin, Schiffer, Basilevitch (contour integration); Schiffer, Lavrentiev, Marty, Biernacki, Schaeffer and Spencer, Grad, Golusin, Springer, Schiffer and Gararbedian (variational methods).

In this general survey, the author presents the results, as far as possible in the chronological order in which they were obtained, and pays the proper attention to matters of priority. He makes a special effort to stress the importance of Grötzsch's contributions, that do not seem to have found the recognition they deserve. Some results concerning subclasses of S, such as convex (Study, Gronwall, Löwner), starlike (Alexander, Nevanlinna), uniformly bounded (Pick), real and typically real (Dieudonné, Rogosinski, Ssasz), and odd (Littlewood and Paley, Fekete, Szegö) univalent functions are also indicated. The theory of univalent functions may be extended to p-valent functions, by elementary methods (Biernacki, Spencer), but especially by the method of symmetrization (Steiner, Faber, Hayman, Beurling, Polya and Szegö). Yet, as already mentioned, the monograph has as its Leitmotiv the GCT (General Coefficient Theorem). This theorem, already obtained by the author in 1954 (Trans. Amer. Math. Soc. vol. 77 (1954) pp. 262–280), in essentially its present form, may be considered as an application of ideas due to Teichmüller (Sitzungsberichte der Preuss. Ak. Wiss., Phys.-Math. Kl., 1938, pp. 363–375; see also Abh. Preuss. Ak. Wiss., Math.-Nat. Kl., 1939, no. 22). These in turn were suggested by the work of Grötzsch, who had obtained most of the results known (at that time) and some new ones in the theory of univalent functions, by a consistent use of his method of strips, a form of the extremal metric method. In the book under review, the statement of the GCT takes an entire page—and even this "conciseness" is made possible only because the statement itself is preceded by four definitions and a lemma, whose combined statements, in turn, take almost another two pages. Enunciated in this way, the GCT may impress one as somewhat cumbersome—especially as its great generality is rarely used to the full. However, this theorem more than justifies the effort needed in its statement and proof, by permitting one to obtain in a beautifully unified way a very large proportion of all results known at present in the theory of univalent functions. In a drastically simplified (therefore not completely accurate) formulation, the GCT reads as follows: Let $d\zeta^2 = Q(z)dz^2$ be a quadratic differential, defined on a finite, orientable Riemann surface $\mathfrak{R}$ and positive or zero on any boundary $\mathfrak{R}$ may have. The trajectories of $d\zeta^2$ (i.e., maximal regular curves on $\mathfrak{R}$, on which $d\zeta^2 > 0$) that are closed, or end at zeros, or simple poles of $Q(z)$, split $\mathfrak{R}$ into a family $\{\Delta\}$ of domains Δ_j. Furthermore, consider a family $\{f\}$ of functions $f_j(z)$ satisfying certain conditions, insuring that they are essentially homeomorphisms on Δ_j into $\mathfrak{R}$, homotopic to the identity and leaving invariant the poles of $Q(z)$ inside the Δ_j's. If A is a double pole of $d\zeta^2$ and selecting the local uniformizing parameter so that A should be represented as the point at infinity, $Q(z) = \alpha z^{-2} + \beta z^{-3} + \cdots$, and each $f(z)$ has an expansion $f(z) = az + a_0 + a_1 z^{-1} + \cdots$; if A is a pole of order $m > 2$, then $Q(z) = \alpha z^{m-4} + \beta z^{m-5} + \cdots$, and $f(z) = z + a_{m-3} z^{3-m} + \cdots$. Under these conditions the GCT states that $\text{Re}\left\{\sum \alpha \log a + \sum \alpha a_{m-3}\right\} \leqq 0$, where the first sum is extended over all double poles of $d\zeta^2$ and the second sum over the poles of order higher than two. Equality can occur only under certain well defined conditions; hence, the theorem leads also to the determination of the extremal mappings. As in the author's quoted paper of 1954, the proof of the GCT (and also of an extension, where the positivity of $d\zeta^2$ on the boundary of $\mathfrak{R}$ is dropped) makes essential use of the properties of the conformally invariant metric $|d\zeta| = |Q|^{1/2}|dz|$. The GCT and its proof make up the content of the fourth chapter, being preceded by two preparatory chapters (Chapter II on modules and extremal length and Chapter III on quadratic differentials) and followed by three chapters of applications of the GCT.

In Chapter V, by very simple applications of the GCT (e.g., with $d\zeta^2 = -dz^2/z^2$ and $\{\Delta\}$ consisting of a single domain D), one obtains Koebe's classical results concerning canonical mappings onto slit domains. Further results of the same general nature due to Hilbert and Grötzsch are similarly obtained. In Chapter VI, using $d\zeta^2 = dw^2/w^2(w-1/4)$, the famous first theorem of Koebe and also the value $k = 1/4$ of the Koebe constant are obtained in a few lines, followed by similar short proofs for many other classical results, such as: (i) If $f \in S$, $|z| = r$, $0 < r < 1$, then $r(1+r)^{-2} \leqq |f(z)| \leqq r(1-r)^{-2}$, with equality only for $f(z) = z(1+e^{i\alpha}z)^{-2}$, α real, $z = \pm re^{-i\alpha}$; (ii) If $f \in \Sigma$, $|z_1| = |z_2| = \rho > 1$, $z_1 \neq z_2$, then $|(f(z_1) - f(z_2))/(z_1 - z_2)| \geqq 1 - \rho^{-2}$; (iii) The GCT with $d\zeta^2 = dw^2/w^2(w-d)^2$ and for Δ the image of $|z| < 1$

under $f(z)=z(1+z)^{-2}$, leads to $|f'(re^{i\theta})| \geqq (1-r^2)r^{-2}|f(re^{i\theta})|^2$ for any meromorphic function, univalent in $|z|<1$, with $f(0)=0$, $f'(0)=1$; equality can occur only for $f(z)=z(1+cz+e^{2i\theta}z^2)^{-1}$ (c arbitrary, θ real). This, together with (i), permits to obtain Koebe's distortion theorem and many other similar results. Follow next some (known) diameter theorems, results concerning regions of values for coefficients of univalent functions (examples: (i) if $f\in\Sigma$, then $|a_1|\leqq 1$ with $a_1=e^{2i\theta}$ only for $f(z)=z+c+e^{2i\theta}z^{-1}$; (ii) if $f\in S$, then $|A_2|\leqq 2$, with $A_2=-2e^{i\alpha}$ (α real) only for $f=z(1+e^{i\alpha}z)^{-2}$); bounds for $|\log f'(re^{i\theta})|$, $|f''(z)/f'(z)|$ and $|\log (zf'(z)/f(z))|$ and some original results of the author. The chapter ends with Teichmüller's coefficients' inequality. Chapter VII contains two very general theorems on families of univalent functions, due to the author. They contain as corollaries such theorems as: Let $f(z)$ be regular (not necessarily univalent) for $|z|<1$, $f(0)=0$ and either $f(z_1)f(z_2)\neq 1$ or $f(z_1)\overline{f(z_2)}\neq -1$ holds for $|z_1|$, $|z_2|<1$; then $|f'(0)|\leqq 1$, with equality only for $f=e^{i\theta}z$, θ real.

The eighth and last chapter is somewhat different from the others, in that it refers to multivalent rather than to univalent functions. The GCT is not directly applicable, but Teichmüller's principle can be used to obtain a quadratic differential associated with the problem, and the method of symmetrization is successfully used. Many of the results quoted here were first obtained by Hayman, whose recent book on p-valent functions should be consulted by anybody interested in the subject. If F_p stands for the class of functions $f(z)=z^p+a_{p+1}z^{p+1}+\cdots$ circumferencially p-valent and regular in $|z|<1$, then Hayman has shown that Koebe's first theorem (with constant 1/4) and the bounds $r(1\pm r)^{-2}$ for $|f(z)|$ hold also for $f\in F_1$, with the same extremal functions as for $f\in S$. From this and the remark that $f\in F_p$ implies $f^{1/p}=z+p^{-1}a_{p+1}z^2+\cdots\in F_1$ many further results follow, of which the following are characteristic: (i) if $f\in F_p$, $|z|=r$, $0<r<1$, then $(r(1+r)^{-2})^p\leqq|f(z)|\leqq(r(1-r)^{-2})^p$; (ii) if $f\in F_p$, then $|a_{p+1}|\leqq 2p$; (iii) for $f\in F_p$, $|pa_{p+2}-2^{-1}(p-1)a_{p+1}^2|\leqq 3p^2$, with equality only for $f(z)=(z(1+e^{i\alpha}z)^{-2})^p$; hence, using (ii), $|a_{p+2}|\leqq 2p^2+p$ and, in particular, for $p=1$, $|a_3|\leqq 3$.

It is esthetically most satisfying to see so many results, originally obtained by a diversity of methods, to follow so naturally from a single principle (Teichmüller's), most of them even by the direct use of a single theorem (the author's GCT). It is also a testimony to the power of the method that such a wealth of results can be presented mostly with complete proofs, within 160 pages. The exposition is clear, the style sometimes reminiscent of that of a paper rather than a book. The printing is very good. No serious mistakes came to the reviewer's attention. A few trivial ones may be mentioned. There are some typographical errors (e.g., on p. 21 in the column title; on p. 24, 3rd line from the bottom, read

$$\frac{1}{2\pi}\log R \quad \text{for} \quad \log\frac{1}{2\pi}R;$$

p. 159, 10th line from the bottom, first + should be a −, etc.). Also, it seems to the reviewer that on p. 78 the paragraph following Theorem 5.9 should precede the statement of the theorem; finally, on p. 30, the configuration of Figure 3 refers to a pole of order 5 rather than to one of order 3. Needless to say, such errors (hardly noticeable to the casual reader, easily corrected by the careful one) do not detract anything from the value of an otherwise excellent book.

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