

level, and as such represents an effort on the part of the author which all too few mathematicians undertake nowadays.

As regards its mathematical content, the unifying theme is indicated exactly by the title of the monograph; the main outlines of the historical development of the notion of statistical independence are laid out, and it is shown how this notion has proved to be a keystone of analysis and number theory, as well as of probability and statistics, with which it is more usually associated. With a view to kindling interest by making the underlying ideas more accessible, the author has chosen to omit some details, but he gives a bibliography adequate for leading the interested reader back to the literature. Some knowledge is supposed of Lebesgue measure and integration, Fourier integrals and number theory.

Most of the development hinges ultimately on the specific realization of statistical independence provided by the Rademacher functions. The range of problems considered is very broad, including continued fractions, the law of large numbers and the central limit theorem, normal numbers, prime numbers and additive number-theoretic functions, the ergodic theorem, and the convergence of series with random signs. None of these topics is treated at all intensively, but the rich flow of ideas, the many interrelations which are brought out, and the elegance of exposition, all contribute to provide a remarkable panoramic view of one mathematical landscape.

Professor Kac is an ardent exponent of the theory that what is newest is not always what is best, and he takes the opportunity here to argue against what he considers overemphasis on abstraction in modern mathematics. This is first-class hortatory writing, and it should be read by every graduate student, along with Bourbaki.

W. J. LEVEQUE

Formes sesquilineaires et formes quadratiques. By N. Bourbaki. *Éléments de Mathématique I, Livre II. Actualités Scientifiques et Industrielles*, no. 1272, Paris, Hermann, 1959. 211 pp.

This is volume 24 of Bourbaki's *Elements* (in the simple minded numbering system that seems to serve better than calling it Chapter 9 of Book 2 of Part I). From an impetuous youth who dared to announce that he planned to write up all of mathematics, N. Bourbaki has turned into a middle aged fixture gallantly and interminably teaching us how to do things right. Addicts expect more from Bourbaki—and they get it: a text ranging from watery soup to several solid meat courses, with a stunning collection of exercises for hors

d'oeuvres. (Cash customers should note that the exercises take up nearly a third of the text.) All this and a fourteen page historical note up to the usual erudite standard.

The volume begins with an appropriate abstract blast, material such as orthogonal subspaces, rank, change of rings etc. being discussed in the context of an arbitrary pairing of a left module and a right module to a two-sided module. But before long things have simmered down to the familiar hermitian and skew-hermitian forms. Witt's cancellation theorem takes the place of honor it deserves and gets a well organized proof. In time honored fashion the individual cases are then studied, with alternate forms coming first. Bourbaki does not care that the canonical form for alternate matrices over an integral domain requires only that finitely generated ideals be principal (and who can blame him?). Topics associated with the principal axis theorem are given a thorough workout, with ample attention to normal linear transformations and quaternion coefficients. Witt's ring of quadratic forms is introduced but at present it leads nowhere. Clifford algebras get an excellent detailed exposition. And now—hold your breath—we are ready for angles and trigonometric functions. The context at first is an arbitrary quadratic form over an arbitrary field of characteristic $\neq 2$, but after a stubborn rearguard action Bourbaki retreats to a real closed field.

I have already praised the exercises. Let me add that they could have been collected only by a person (people?) who had thoroughly mastered the subject and its voluminous literature. The following list is only a sample: the orders of the finite classical groups, isomorphisms between classical groups of low dimension, inversive geometry, Wiegmann's theorem on products of normal matrices, a generalization of Mackey's theorem on dual vector spaces of countable dimension, a quadratic form in $n+1$ variables over K represents 0 if K is not formally real and $K^*/(K^*)^2$ has order n , quadratic forms over a discrete valuation ring, Frobenius rings, the Cayley transform, the elementary divisors of linear transformations preserving a form, the factorization of an orthogonal transformation into symmetries, ruled quadrics, and the peculiar fields for which the principal axis theorem is valid (I can't omit that one; see page 132).

A reader who slugs his way through the book, including most of the exercises, will find himself thoroughly educated on the basic theory of quadratic forms. Of course the arithmetic theory is another matter. I presume that this is not a "fundamental structure," so we must wait patiently.