## BOOK REVIEWS

*A collection of mathematical problems.* By S. M. Ulam. Interscience Tracts in Pure and Applied Mathematics, no. 8. New York, Interscience, 1960. 13+150 pp. $5.00.

This book is unique. There are other collections of problems, for instance, the well-known *Aufgaben und Lehrsätze* of Polya and Szegö, and Knopp's problem book, but these are collections of problems with solutions. The present book is a collection of *unsolved* problems, or at least ones for which the author does not know the answers. There is a closer parallel with Hilbert's famous lecture on mathematical problems at the Paris Congress of 1900 (this Bulletin, vol. 8, pp. 437–479). In comparison, the present collection is less pretentious and more personal. The author does not pretend to forecast the lines of future development of mathematics. He does not even claim that the problems he proposes are central; merely that they reflect his personal interests. They consist of open questions that have arisen from his work in many different fields.

The problems considered are in the spirit of the so-called *Scottish Book*. During the 1930's a notably gregarious group of mathematicians in Lwów, Poland—including Banach, Steinhaus, Mazur, Orlicz, Schauder, Schreier, Ulam, and others—were accustomed to meet for long mathematical discussions in "The Scottish Coffee Shop." From time to time, problems which they posed to each other were written down in a notebook which was kept there for the purpose. (Sometimes the proposer would indicate his estimate of the difficulty of a problem by offering a prize for its solution; perhaps a bottle of wine, or two small beers!) Visiting mathematicians too were invited to add their problems to the collection. After the war this book was carried to Wrocław, where the tradition was revived by some of the surviving members of the group. Many problems from the "New Scottish Book" have appeared in the problem section of *Colloquium Mathematicum.* A few years ago, Ulam circulated privately a translation made from a copy of the original Scottish Book. The interest aroused by this encouraged him to write the present book. The problems include many which he first inscribed in the Scottish Book, but a greater number stem from later years. In fact, the book constitutes a kind of mathematical autobiography. Each of the various fields in which the author has worked has contributed its share of problems. But despite their diversity there is an underlying unity. As the author puts it, "the motif of the collection is a set-theoretical point of view and a combinatorial approach to problems."

The problems are arranged in a logical order which is also roughly chronological. First come problems in set theory: product isomorphisms, projective algebras, logic, and abstract measure theory. Some interesting ideas concerning the existence of non-measurable projective sets are expressed here. The second chapter contains problems about groups and semi-groups. Then come problems about metric and topological spaces, problems concerning various kinds of invariance. There is a short chapter devoted to topological groups, especially the group $S_\infty$ of all permutations of the integers, but actually notions arising from topological groups pervade much of the book. The chapter on analysis is largely concerned with functional equations, especially ones that are only approximately satisfied, and with questions of conjugacy of functions and transformations. The last two chapters, which constitute almost half of the book, are devoted to problems suggested by physics and by computing machines. Some very suggestive ideas concerning the possible role of actual infinities in physical theories, and the relevance of the notions of Cantorian set theory, are expressed. Questions concerning flows in phase space, and the topology of magnetic lines of force, are formulated. The final chapter "makes propaganda" (as the author would say) for the use of computing machines as a heuristic aid. Various conjectures are expressed concerning games, number theory, functional equations, and physical models, that can be tested by Monte Carlo methods. Partial answers obtained by the author and his collaborators are described. In some situations a combination of operator and machine can be more effective than a fully programmed machine. For example, by watching a display produced by the machine, the operator may be in position to steer it toward the location of a critical point more efficiently than would a search code. In such collaboration of machine and operator the author sees the most likely direction of progress in the immediate future.

The problems are neither numbered nor displayed. Some are completely specific, others merely outlined. The author considers that most are of "medium difficulty," but says that he would not be surprised if some should turn out to admit trivial solutions. The reader should perhaps be warned that a few problems are somewhat carelessly stated. For instance, the property of Baire is incorrectly defined on pages 11, 17, and 24; while on pages 19 and 23 the strong and weak properties are not distinguished from each other. On page 58 the discrete topology is classified as not locally compact. But in all these cases it is easy enough to see what was intended.

In conclusion, this is not a book to read through at one sitting, nor is it one to plough through like a textbook. The book is valuable not only for the problems which is contains, but also for the glimpse it affords of a fertile mathematical imagination at work, and for the problem-centered approach to mathematics which it fosters.

JOHN C. OXTOBY

*Axiomatic set theory*. By Patrick Suppes, Princeton, Van Nostrand 1960. xii+266 pages. $6.00.

There has long been a need for a textbook on axiomatic set theory, and Professor Suppes' book meets this need splendidly. It is based upon the Zermelo-Fraenkel-Skolem axioms, which use $\epsilon$ and $O$ as the only primitive non-logical symbols. There is only one kind of variable, referring to both sets and individuals. The notion of set and corresponding restricted variables are introduced by means of the definition: $y$ is a *set* if and only if $(\exists x)(x \in y \vee y = 0)$. Logical notation is employed, and the grammar of the theory is presented in a rigorous way, but no special results of symbolic logic are needed. For the most part, the book can be read in the same way as a book on any other mathematical theory.

Before discussing various special points, we give an abbreviated table of contents. CHAPTER 1. Introduction: Set Theory and the Foundations of Mathematics. Logic and Notation. Axiom Schema of Abstraction and Russell's Paradox. More Paradoxes. CHAPTER 2. Formulas and Definitions. Axioms of Extensionality and Separation. Intersection, Union, and Difference of Sets. Pairing Axiom and Ordered Pairs. Definition by Abstraction. Sum Axiom and Families of Sets. Power Set Axiom. Cartesian Product of Sets. Axiom of Regularity. CHAPTER 3. Relations and Functions. CHAPTER 4. Equipollence. Finite Sets. Cardinal Numbers. Finite Cardinals. CHAPTER 5. Definition and General Properties of Ordinals. Finite Ordinals and Recursive Definitions. Denumerable Sets. CHAPTER 6. Fractions. Non-Negative Rational Numbers. Rational Numbers. Cauchy Sequences of Rational Numbers. Real Numbers. Sets of the Power of the Continuum. CHAPTER 7. Transfinite Induction and Definition by Transfinite Recursion. Elements of Ordinal Arithmetic. Alephs. Well-Ordered Sets. CHAPTER 8. Some Applications of the Axiom of Choice. Equivalents of the Axiom of Choice. Axioms Which Imply the Axiom of Choice.