discrete topology is classified as not locally compact. But in all these cases it is easy enough to see what was intended.

In conclusion, this is not a book to read through at one sitting, nor is it one to plough through like a textbook. The book is valuable not only for the problems which is contains, but also for the glimpse it affords of a fertile mathematical imagination at work, and for the problem-centered approach to mathematics which it fosters.

JOHN C. OXTOBY

Axiomatic set theory. By Patrick Suppes, Princeton, Van Nostrand 1960. xii+266 pages. $6.00.

There has long been a need for a textbook on axiomatic set theory, and Professor Suppes' book meets this need splendidly. It is based upon the Zermelo-Fraenkel-Skolem axioms, which use ϵ and O as the only primitive non-logical symbols. There is only one kind of variable, referring to both sets and individuals. The notion of set and corresponding restricted variables are introduced by means of the definition: y is a *set* if and only if $(\exists x)(x \in y \vee y = 0)$. Logical notation is employed, and the grammar of the theory is presented in a rigorous way, but no special results of symbolic logic are needed. For the most part, the book can be read in the same way as a book on any other mathematical theory.

Before discussing various special points, we give an abbreviated table of contents. CHAPTER 1. Introduction: Set Theory and the Foundations of Mathematics. Logic and Notation. Axiom Schema of Abstraction and Russell's Paradox. More Paradoxes. CHAPTER 2. Formulas and Definitions. Axioms of Extensionality and Separation. Intersection, Union, and Difference of Sets. Pairing Axiom and Ordered Pairs. Definition by Abstraction. Sum Axiom and Families of Sets. Power Set Axiom. Cartesian Product of Sets. Axiom of Regularity. CHAPTER 3. Relations and Functions. CHAPTER 4. Equipollence. Finite Sets. Cardinal Numbers. Finite Cardinals. CHAPTER 5. Definition and General Properties of Ordinals. Finite Ordinals and Recursive Definitions. Denumerable Sets. CHAPTER 6. Fractions. Non-Negative Rational Numbers. Rational Numbers. Cauchy Sequences of Rational Numbers. Real Numbers. Sets of the Power of the Continuum. CHAPTER 7. Transfinite Induction and Definition by Transfinite Recursion. Elements of Ordinal Arithmetic. Alephs. Well-Ordered Sets. CHAPTER 8. Some Applications of the Axiom of Choice. Equivalents of the Axiom of Choice. Axioms Which Imply the Axiom of Choice.

The Axiom of Regularity: $A \neq 0 \rightarrow (\exists x)[x \in A \ \& \ (\forall y)(y \in x \rightarrow y \notin A)]$ forms part of the axiom system. This axiom has as a consequence the exclusion of infinitely descending ϵ-sequences. Though it simplifies the definition of ordinal number and seems to be a plausible assumption, it should be noted that it is not necessary for the development of set theory.

In order to develop the theory of cardinal numbers, the author uses a special operation $K(A)$ [read: the cardinal number of A], with the axiom: $K(A) = K(B)$ if and only if there is a 1-1 correspondence between A and B. Given the axiom of choice, this may be avoided, since $K(A)$ can be defined as the smallest ordinal equipollent with A. As Dana Scott has noted, the axiom of regularity also enables us to define $K(A)$, but the definition is too complex to be given in an elementary text.

The elegant unorthodox treatment of finite sets by means of Tarski's definition[1] allows the author to develop the theory of finite sets before the theory of finite ordinals. However, the traditional approach would have been more natural, and appreciably easier for the beginner to follow.

The chapter on the definition of the real numbers is carefully and lucidly done. Inclusion of this chapter makes the book quite suitable for use in courses on the Fundamental Concepts of Mathematics.

In the fine development of the theory of ordinals, particularly praiseworthy is the explicit use of Hartog's function[2] $H(A)$ for the definition of the alephs. There is also a good selection of equivalent forms of the axiom of choice, including the well-ordering theorem, law of trichotomy for cardinals, Zorn's lemma, Hausdorff's maximal principle, and the Teichmüller-Tukey lemma.

To allow for the existence of cows and molecules and other extra-mathematical objects, the system used in the text permits the existence of individuals, i.e. objects which contain no sets and are not themselves sets, though such individuals are strictly unnecessary for mathematical purposes. Also, in contrast to the von Neumann-Bernays-Gödel type of system (as given, for example, in Gödel's *Consistency of the continuum hypothesis*) there are no proper classes, i.e. objects which contain sets but are not themselves sets. This makes the system conceptually simpler, but complicates a good part of the work on definition by transfinite recursion. Some interesting assertions (such as the existence of a universal choice function or a well-

[1] A is finite if and only if every nonempty family of subsets of A has a maximal element.

[2] $H(A)$ is the least ordinal which is not equipollent with a subset of A.

ordering of the universe) cannot even be made, since they involve proper classes. However, it has been shown that, in a certain sense,[3] the two systems are equivalent, so that the choice of a system is primarily a matter of taste. In any case, Professor Suppes' book will take its place as the most usable text on axiomatic set theory, and should be a model of lucidity for future textbook writers.

ELLIOTT MENDELSON

BRIEF MENTION

Bevezetés a hálóelméletbe (*Introduction to lattice theory*). By Gábor Szász. Budapest, Hungarian Academy of Sciences, 1959. 225 pp. (Hungarian).

This is a textbook (complete with exercises) on the first-year graduate level; the prerequisites are the bare elements of set theory, algebra, and topology. The exposition is detailed and polished. Table of contents: I, Partially ordered sets; II, Generalities on lattices; III, Complete lattices; IV, Distributive and modular lattices; V, Some special subclasses of the class of modular lattices; VI, Boolean algebra; VII, Semi-modular lattices; VIII, The ideals of a lattice; IX, Congruence relations.

Theory of differential equations. By A. R. Forsyth. New York, Dover, 1960. 13+340 pp., 11+344 pp., 10+391 pp., 16+534 pp., 20+478 pp., 13+596 pp. $15.00 (set of six vols. bound as three).

The six volumes of this old (1890) standard treatise are reprinted unabridged, combined in pairs into three volumes.

Proceedings of the International Congress of Mathematicians, 1958. Ed. by J. A. Todd. New York, Cambridge University Press, 1960. 64+573 pp. $12.50.

This volume contains the official record of the International Congress held in Edinburgh in August, 1958. It includes the complete texts of 17 one-hour and 33 half-hour invited addresses, and also a listing by title of the short communications made by members of the Congress.

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[3] Every sentence of ZFS set theory which is provable in NBG set theory is provable also in ZFS set theory.