$$\exp\left[i\int \phi(t+\tau)dx(\tau,\alpha)\right]$$

and

$$\exp\left[i\,\epsilon\int\int K(t+\tau_1, t+\tau_2)dx(\tau_1,\alpha)dx(\tau_2,\alpha)\right].$$

These are expanded in orthogonal series and their covariances and spectra determined (it should be noted that some of these results can be found easily by standard methods). In two other lectures the author applies his method to the analysis and synthesis of a four-terminal electrical network. An empirical analysis is sketched which involves the ingenious device of generating Laguerre polynomials electrically with lattice networks, to use as kernels for the polynomial functionals.

This book provides some rather novel methods of attack on a class of difficult problems. In the reviewer's opinion, it should prove to be stimulating and useful reading for a considerable group of applied mathematicians, engineers, and physicists.

WILLIAM L. ROOT

Information theory and statistics. By Solomon Kullback, New York, John Wiley and Sons, Inc., 1959. 18+395 pp. $12.50.

Kullback and Leibler elaborated a definition of the information contained in an experiment to distinguish between two hypothetical distributions on a sample space. The original definition was given by Wiener in his *Cybernetics* and is formally a generalization of the one by Shannon, which is motivated by communication theory considerations and is justified by the nontrivial coding theorem for channels. The book under review does not attempt to generalize Shannon's theory but is concerned with the properties of the generalized information measure as a statistic in testing of hypotheses.

SEYMOUR SHERMAN

Probability and related topics in physical sciences. By Mark Kac, with special lectures by G. E. Uhlenbeck, A. R. Hibbs and Balth. van der Pol. Lectures in Applied Mathematics, vol. 1. New York, Interscience, 1959. 13+266 pp. $5.60.

What is the probability that a mathematics book (chosen at random?) will be informative, clearly written, and delightful to read? Although Professor Kac does not consider this question in his recent book, he clearly demonstrates that the set of such books is nonempty.

This is an expanded version of a series of lectures by the author delivered at the Seminar in Applied Mathematics held in Boulder, Colorado in the summer of 1957. The present volume is the first of four which will contain the proceedings of this seminar. In addition to the author's lectures, this volume contains four appendices with special lectures by G. E. Uhlenbeck, A. R. Hibbs, and Balth. van der Pol. The adjective "charming" is the only one which adequately describes this book. First, the informal style of the lectures has been retained. Secondly, the book summarizes some of the author's best research. His distaste for general theories has led the author to base the presentation on a series of special (but nontrivial) examples. This approach has produced an unusual clarity with which the concepts and fundamental ideas are presented.

Chapter I deals with the nature of probabilistic reasoning. For the main part, the author is interested in showing how useful probability models can be constructed and applied to a variety of problems. Each model discussed is chosen with a special point in mind. For example, in his analysis of "the average number of real roots of an algebraic equation" the author shows how a statistical treatment may be of value in cases where individual treatment is impossible, thus hinting at the method of Gibbs' ensembles.

Chapter II is devoted to illustrating the standard techniques of probability theory. Included are a few unusual methods due to the author, e.g. proving a limit theorem by perturbation techniques. There is a good treatment of the polymer chain problem (A chain of n links in space, each link forming a fixed valence angle with the previous link, is given. Find the distribution of the size of the chain.) and a fair treatment of the simple random walk problem. Combinatorial methods are also discussed, and the interplay between combinatorics and analytics is mentioned. However, the latter material does not have any real value for the rest of the book.

Chapter III, on the role of probability in classical statistical mechanics, is a *masterpiece*. There are two questions which the author poses for discussion:

(1) "Is it possible to reconcile both time reversibility and recurrence with 'observal' irreversible behavior?

(2) "Is it possible to achieve such a reconciliation in the realm of classical mechanics?"

Interest and importance are added to these questions by a summary of the views of Boltzmann, Gibbs, and Ehrenfest. Also included is an account of the Boltzmann-Zermelo controversy and an account of the Poincaré recurrence theorem on which this controversy is based. The author considers first the relatively simple Ehrenfest diffusion model and then turns to more complicated examples that he himself invented. The discussion is very enlightening even though neither question is completely answered. One model, the Kac "master equation" approach to Boltzmann's theory for a monatomic gas, deserves special mention. This is the most physically meaningful example in terms of which the above questions are considered. It is also a medium for a display of the author's mathematical power. Unfortunately, as the author himself admits, the example raises as many questions as it settles. Chapter III ends with a description of Smoluchowski's analysis of the Svedberg colloid experiment (Brownian motion) and the Smoluchowski processes (generalized Poisson processes). On page 45 the secret of the success of Chapter III is revealed in the author's own words: "There can be no denying that our reaction to a scientific statement depends greatly on the context in which it is made. In another context and in a suitable language it may be highly revealing and suggestive." Professor Kac takes every opportunity to make his models "revealing and suggestive," with an enthusiasm that abounds on every page.

Chapter IV, on integration in function space, is a collection of results on the evaluation of the functional

$$E_W\left\{\exp\left(-\int_0^t V(\xi + x(\tau))d\tau\right) \middle| \, x(t) = 0\right\}$$

(W is the Wiener process) with applications to mathematical and physical problems. The asymptotic distribution of eigenvalues of a Schrödinger-like equation is discussed as well as the evaluation of a quantum mechanical partition function. The final section summarizes the author's contributions to probability in potential theory. It is interesting to note how many fundamental ideas in this area were suggested to the author by physical considerations.

Three of the four appendices are essentially unrelated to the main part of the book. Appendix II, by A. R. Hibbs, is a discussion of the probability amplitude function in quantum mechanics. Appendices III and IV, by Balth. van der Pol, are concerned with "unsmoothing" smoothing operators and with finite difference analogues for the wave and potential equations.

Only Appendix I, by G. E. Uhlenbeck, has any significant bearing on the main part of the book. Uhlenbeck is concerned with the two questions mentioned previously and the attempts which have been made to answer them. In the preface the author refers to Chapter III as a "running commentary" on the first part of Appendix I. It is amusing, therefore, to view Appendix I as a scale against which Kac's achievements can be measured. When Uhlenbeck mentions the problem of finding "appropriate laws for the approach to equilibrium" or of relating the classical Liouville approach for the description of a gas to the master equation, the reader will find comfort in a more thorough discussion of these points in the "ring" model and the Kac monatomic gas model of Chapter III. On the other hand, when Uhlenbeck mentions the Bogoliubov approach to the description of a gas, the corresponding feeling of comfort is not to be found. Uhlenbeck's discussion is good, but quite intuitive (rather than rigorous) and a bit brief at times. To the reviewer, the real issue is to define the problems in a more reasonable form. Because of this problem of definition, Appendix I may well give the reader a much deeper appreciation for the accomplishments of Kac in Chapter III. At the same time the reader will discover that Kac and Uhlenbeck talk a somewhat different language. In spite of their many years of association, a large gap still exists between their viewpoints.

An excellent set of notes and bibliography has been included at the end of the book. Readers interested in the history of the problems discussed will find these notes add a certain glamor to the various sections.

There are several groups to whom this book is highly recommended. All persons interested in probability theory will find the models and examples presented well worth their attention. The physicist with prior knowledge of probability theory, who is interested in applications, will find this an excellent exposition of the methods and techniques involved. In summary, for all those interested in probability and related topics in physical science, the book, will serve as a concise and elegant summary of the author's own investigations into this important borderline area.

GLEN BAXTER

Homotopy theory. By Sze-Tsen Hu. New York and London, Academic Press, 1959. xiii+347 pp. $11.00.

Although homotopy theory has been rather intensively studied for 25 years and is today one of the principal branches of algebraic topology, this is the first textbook on the subject. (A possible exception to this statement is the Cambridge Tract by P. J. Hilton entitled *An introduction to homotopy theory*; however, this booklet is only about 140 pages long and is principally concerned with certain special topics.) Since this book by Hu is the only text in this large field, its acquisition is a "must" for any mathematical library having any