generating systems with the property that each subsystem with the same cardinality is also a generating system, universal homomorphic images, and universal subgroups.

Among many notable features of this book which should be mentioned are the excellent bibliography, the exercises with a wide range of difficulty which cover virtually every topic presented, and the statement of eighty-six unsolved problems which already have led to new contributions to the theory of abelian groups. The book is printed in the same large clear type and format as the Hungarian mathematical journals and is remarkably free of misprints.

This book is an important addition to mathematical literature and is highly recommended to anyone whose interests touch the theory of abelian groups.

R. A. BEAUMONT

Special functions. By Earl D. Rainville. New York, Macmillan, 1960. 12+365 pp. $11.75.

This work of 365 pages and 21 chapters introduces the reader to a large number of "special" functions and their properties, and with this purpose (the author informs us) the material of the book has been the basis of a course given by him since 1946. Most of these functions are classical: the Gamma function, Hypergeometric function, Bessel functions, Elliptic functions (including the Jacobi elliptics), and the important orthogonal polynomials.

With regard to these long-known and deeply studied functions one merit of the book lies in bringing under one cover, at less than encyclopedic length, this large variety of important tools of classical analysis. There also are results on generating functions that are perhaps not well known, so that one comes upon relatively new material; and in addition the book discusses, in some detail, Generalized hypergeometric functions, with special reference to polynomials defined in terms of these. Much of this information is not easily accessible elsewhere, and represents a valuable part, and in a certain sense the most interesting part, of the volume.

Chapters 1 and 3 are short introductions to infinite products and asymptotic series; and Chapter 2 discusses the Gamma and Beta functions. Chapters 4 and 5 take up the hypergeometric function and its generalizations. Here one finds the author's own results on contiguous function relations. Chapter 6 treats Bessel functions briefly (there are so many available sources of information for these); and briefly again, 7 touches on the confluent hypergeometric function.

With Chapter 8, on Generating Functions, we enter the region of the author's special interest. If $\{f_n(x)\}$ is an infinite sequence of functions (in particular, of polynomials) and $\{c_n\}$ an arbitrary sequence of nonzero constants, then the formal series $G(x, t) = \sum_{n=0}^{\infty} c_n f_n(x)$ defines a generating function for $\{f_n(x)\}$. Different choices of $\{c_n\}$ give different generating functions, and while $\{f_n\}$ and $\{c_n f_n\}$ are on a par for many purposes (for example, in the problem of expanding functions in terms of $\{f_n\}$), judicious choice of $\{c_n\}$ can yield important information about $\{f_n\}$ by way of properties of the corresponding function $G(x, t)$. In 8 there is a brief discussion of polynomial sets whose generating functions have special form, e.g., $G(2xt-t^2)$; $e^t\psi(xt)$; $A(t)\exp\{-xt/1-t\}$; and the Boas-Buck case $G(x, t)=A(t)\psi[xH(t)]$, which includes some of the others. Also the extension to $G(x, t)=A(t)\psi[xH(t)+g(t)]$.

Chapter 9 gives some of the properties of orthogonal polynomials in general; and in particular, 10, 11, 12, 16 treat the polynomials of Legendre, Hermite, Laguerre and Jacobi. These chapters also include numerous generating functions for the various polynomials, that were obtained by the author and some of his students. Many of the generating functions involve generalized hypergeometric functions. 17 and 18 briefly examine Ultraspherical and Gegenbauer polynomials, certain polynomials of Bateman, Rice, Sister Celine, the Bessel polynomials of Frink and Krall, and some others.

Chapter 13 takes up the A-type classification of polynomial sets that was given by the reviewer, particularly sets of A-type zero (which are subsumed under the Boas-Buck class of sets); and considers an extension of A-type sets obtained by replacing the role of the differential operator D by the operator $\sigma \equiv D\prod_{i=1}^{q}(\theta+b_i-1)$ where b_i is a constant and $\theta \equiv xD$. Many results with σ parallel those using D. In 14 the author describes the technique of Sister Celine which enables one to obtain recurrence relations for many polynomial sets defined by means of general hypergeometric functions. 15 is a brief introduction to the symbolic notation in which many polynomial sets can be written. The last three chapters are given over to Elliptic functions.

The book lists a Bibliography from which the text material can be supplemented; and each chapter ends with a set of Exercises, which serve in part to develop the reader's technique and in part to advance the theoretical content of the book. There are no applications made of the "special" functions treated here, and on reading the book one has the feeling that the author would ask you to believe that these functions are interesting in and of themselves. This accepted, one can further believe that a useful and informative course can be based on the book.

I. M. SHEFFER