Foundations and *Elements*. Only history will tell if one buries the other. Projective methods, which have for some geometers a particular attraction of their own, and which are of primary importance in some aspects of geometry, for instance the theory of heights, are of necessity relegated to the background in the local viewpoint of *Elements*, but again may be taken as starting point given a prejudicial approach to certain questions.

But even more important, theorems and conjectures still get discovered and tested on special examples, for instance elliptic curves or cubic forms over the rational numbers. And to handle these, the mathematician needs no great machinery, just elbow grease and imagination to uncover their secrets. Thus as in the past, there is enough stuff lying around to fit everyone's taste. Those whose taste allows them to swallow the *Elements*, however, will be richly rewarded.

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Foundations of Modern Analysis. By J. Dieudonné. New York, Academic Press, 1960. 14+361 pp. $8.50.

The purpose of this book is to provide the necessary elementary background for all branches of modern mathematics involving Analysis, and to train the students in the use of the axiomatic method. It emphasizes conceptual rather than computational aspects. Besides pointing out the economy of thought and notation which results from a general treatment, the author expresses his opinion that the students of today must, as soon as possible, get a thorough training in this abstract and axiomatic way of thinking if they are ever to understand what is currently going on in mathematical research. The students should build up this "intuition of the abstract", which is so essential in the mind of a modern mathematician. The angle from which the content of this volume is considered is different from the ones in traditional texts of the same level because the author does not just imitate the spirit of his predecessors but instead has a more independent pedagogical attitude. This book takes the students on a tour of some basic results, among them the Tietze-Urysohn extension theorem, the Stone-Weierstrass approximation theorem, the Ascoli compactness theorem, the Jordan curve theorem and the F. Riesz perturbation theory. These are some of the hills in the scenery which are surrounded by nice valleys connecting them. This course, to be taught during a single academic year, is *elementary* in the sense that it is intended for *first year* graduate students or exceptionally advanced undergraduates. Naturally, students must have a good working knowledge of classical Calculus and of elementary Linear Algebra before reading this volume. The book includes a good list of problems, some of them particularly interesting and unusual for a textbook. Specific references to the books of Ahlfors, Bourbaki, Coddington-Levinson, Halmos, Jacobson, Kelley, Loomis and Taylor are included to assist the students in completing their knowledge.

Chapter I (Elements of set theory) treats the indispensable minimum about sets, Boolean algebra, product sets, mappings and denumerable sets. The author does not try to put set theory on an axiomatic basis. He remarks that one very seldom needs more than elementary properties in the applications of set theory to Analysis. The author states the axiom of choice neatly and makes no noise about it. He says that it can sometimes be shown that a theorem proved with the help of that axiom can actually be proved without it. However he never goes into such questions, which properly belong to Logic.

Chapter II (Real numbers) derives the properties of real numbers from a certain number of statements taken as axioms. The real numbers system is presented as an Archimedean ordered field satisfying the nested intervals condition. These axioms can, of course, be proved to be consequences of the axioms of the natural integers together with parts of set theory through the Dedekind or Cantor procedures. Although such proofs have great logical interest, they have no bearing whatsoever on Analysis and teachers should not burden students with them in trying to transmit the spirit of mathematical rigor. This is the right attitude shared by this text.

Chapter III (Metric spaces) constitutes the core of the book, as there is developed in it the geometric language in which we now express the results of Analysis and which has made it possible to reach full generality, besides occasionally supplying the simplest and most perspicuous proofs. As the author says, after some experience the student should be able to acquire the conviction that, with proper safeguards, his own geometric intuition is an extremely reliable guide and that it would be a real pity to limit it to ordinary three dimensional space. This chapter deals in a standard way with continuity, completeness, compactness and connectedness. The completion procedure of metric spaces is not mentioned.

Chapter IV (Additional properties of the real line) includes some elementary properties of the real number system, plus the Tietze-Urysohn extension theorem, which is proved through a known explicit formula peculiar to the metric case.

Chapter V (Normed spaces) and *Chapter* VI (Hilbert spaces) present the elementary geometrical aspects of Banach and Hilbert spaces and also discuss convergent series. Propositions on Banach spaces linked to the notion of Baire category and duality theory are not touched upon. Unwarned readers may find the author a little ungenerous concerning the amount of material in Chapter VI, which looks surprisingly short as compared to what one would expect from the warm praise of Hilbert spaces in the text.

Chapter VII (Spaces of continuous functions), after a few indispensable preliminary considerations, presents in a neat and direct form two of the basic tools of Analysis, namely the Stone-Weierstrass theorem and its application to polynomial and trigonometric approximation and the Ascoli compactness theorem in continuous functions spaces. This is a short and elegant chapter, which presents in a tidy form fundamental material not yet standard in elementary textbooks.

Chapter VIII (Differential Calculus) is beautifully written. The subject matter of the chapter is nothing else but the elementary theorems of Calculus, presented in a manner and generality not yet the vogue in textbooks of comparable level. The author is a partisan of an intrinsic formulation and a geometric outlook on Analysis through use of Banach spaces. Aside from several applications of such a general Calculus, one of the sound motivations for this intrinsic viewpoint is the idea of calculus on a manifold which no young mathematician of nowadays can ignore any longer. The author advises the readers in a fatherly way to assume all vector spaces to be finite dimensional if that gives them an additional feeling of security, but he also stimulates the students to greater courage by adding that this assumption will not make the proofs shorter or simpler. By sticking to the fundamental idea of Calculus, namely the *local* approximation of functions by *linear* functions, successive derivatives $f^p(x_0)$ at a point $x_0 \in A$ of a mapping f of an open subset A of a Banach space E into a Banach space F, are defined to be in the Banach space $\mathfrak{L}_p(E; F)$ of all continuous p-linear mappings of $E^p = E \times \cdots \times E$ (p times) into F. The basic rules of Calculus are proved in this geometric setting and reproduce, of course, classical rules when $E = R^n$ and $F = R^m$ are the spaces of n and m variables. The all-important mean value theorem is proved for vector valued functions in the weak form of an inequality, which corresponds to $|f(b) - f(a)| \leqq |b - a| \cdot \sup_{a \leqq x \leqq b} |f'(x)|$ rather than to the more precise classical form expressed as an equality. For most purposes, indeed, as the author points out, all one needs to know is the inequality formulation. The primitive and integral for functions of a real variable are not deduced from the general theory of Lebesgue integration, which has won a definitive place in Mathematics, nor from Riemann integration, which seems to have already seen its golden period and may become an antiquary item, but only for vector valued functions of real variables with discontinuities of first kind (in an awkward classical terminology), or *regulated* functions according to the author's neologism, that is a function having a limit on the right and on the left at each point. The plausibility of this choice is that integration is easily and intuitively defined for step functions and that a mapping f of a compact interval of the real line into a Banach space is regulated if and only if f is the limit of a uniformly convergent sequence of step functions, which allows one to extend integration by uniform continuity. Since the powerful tools of Lebesgue integration are not needed in a number of important questions, it is perfectly feasible to limit the integration process to a category of functions containing the continuous ones and large enough for elementary purposes. This is what the author does by stopping at regulated functions and so going only halfway to Riemann integration.

Chapter IX (Analytic functions) emphasizes only the general facts for analytic functions of a *finite* number of variables with values in Banach spaces. The cases of real and complex variables are discussed simultaneously, as far as this can be done. The presentation goes up to the Cauchy integral theorem in its usual form. Results based on the Weierstrass preparation theorem are not discussed, so that we have here the theory of analytic functions of several variables only in the elementary sense. An *Appendix to Chapter* IX (Application of analytic functions to plane Topology by Eilenberg's method) is one of the pearls in this text. An irreducible minimum concerning indexes, homotopies and essential mappings leads to elegant proofs of Janiszewski's separation theorem and the Jordan curve theorem. Some students and even mature mathematicians know the statement of the Jordan theorem but have never seen its proof. They even understand that false proofs were given by distinguished mathematicians, including Jordan himself. It is therefore welcome to have the accessible and neat proof in this elementary text, besides those already available.

Chapter X (Existence theorems) deals with procedures linked to the notion of completeness and the method of successive approximations in establishing stability theorems for local homeomorphisms under "slight" perturbations and fixed point theorems. Only the most elementary results of this type are exposed. The subject matter of this chapter has become a classical and fashionable way of introducing students to Functional Analysis, as it requires only a few abstract notions in establishing tangible classical results in a unified way. This chapter deals with the implicit function theorem for functions between Banach spaces, the Cauchy existence theorem for ordinary differential equations in vector valued functions and the Frobenius theorem on the complete integrability of total differential equations between Banach spaces.

Chapter XI (Elementary spectral theory) is the gist of the course, not only because it provides an easy approach to a powerful method of Analysis, namely Spectral Theory, but also because it draws practically on every preceding chapter, showing the students that those abstract techniques were not purposeless generalizations. The flavor of this chapter is that, following Fredholm, compact operators can be viewed as "slight" perturbations of general continuous operators, provided one considers as "negligible" what happens in finite dimensional spaces. After a few elementary properties of spectra of continuous operators, the theory of F. Riesz concerning compact perturbations of an identity operator is developed. Since this theory, for topological vector spaces, has found new geometrical applications other than those devised initially, it is nice to have it presented almost at the start of this chapter with few prerequisites. Compact operators in Hilbert spaces, the Fredholm integral equation and the Sturm-Liouville problem are the next goals of this pretty chapter.

The book is very up to date in terminology, taste and fashion. If a general program of graduate study for mathematicians is to be considered, it should be such that students are expected to get familiar with the content of this volume, whatever their future field of specialization may be. Many opinions it contains are stated in an incisive way, well-known to people personally acquainted with the author, in an attempt to eliminate some vicious attitudes repeated over and over again in traditional texts. This is a most valuable elementary book written by a distinguished mathematician which undoubtedly will help to attract fresh talent into Mathematics. In west Europe, in Japan and in the United States, it is not yet as common to have genuinely good elementary texts written by outstanding mathematicians as it is nowadays in Russia, where, in spite of printing costs, inexpensive editions make such books accessible to the pocket of almost every student.

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