ON THE PRIME IDEALS OF SMALLEST NORM IN AN IDEAL CLASS mod $\mathfrak{f}$ OF AN ALGEBRAIC NUMBER FIELD

BY G. J. RIEGER

Communicated by I. J. Schoenberg, February 3, 1961

In 1947, Linnik [3] proved the following theorem:

THEOREM (OF LINNIK). *There exists an absolute constant c such that in every prime residue class mod k there is a prime number p with* $p < k^c$.

A simplified proof of this theorem was given by Rodosskii [7] whose proof (similar to Linnik's) rests basically on (A) function-theoretic lemmas, (B) theorems on L-functions, (C) estimates of character sums, and (D) a sieve method. The theorems (B) can be classified and characterized as follows:

(B1) order of magnitude of the L-functions [5, Chapter 4, Satz 5.4],

(B2) existence of at most one exceptional zero [5, Chapter 4, Satz 6.9],

(B3) Siegel's theorem on the exceptional zero [5, Chapter 4, Satz 8.1],

(B4) functional equation of the L-functions [5, Chapter 7, Satz 1.1],

(B5) number of zeros in vertical strips [5, Chapter 7, Satz 3.3],

(B6) explicit formulae [5, Chapter 7, Satz 4.1, Satz 6.1].

Recently, I have been able to prove the following generalization of Linnik's theorem which I had conjectured elsewhere [6, p. 168]:

THEOREM 1. *For every algebraic number field K there exists a constant c(K), depending on K only, such that in every ideal class* mod $\mathfrak{f}$ *(in the narrowest sense) there is a prime ideal* $\mathfrak{p}$ *with* $N\mathfrak{p} < N\mathfrak{f}^{c(K)}$.

The skeleton of the proof of Theorem 1 can be taken from Rodosskii's proof; the lemmas (A) are the same; the generalized theorems (B1) resp. (B3) resp. (B4) resp. (B5) resp. (C) resp. (D) can be found in [1] and [4] resp. [4] resp. [1] resp. [1] resp. [2] resp. [6]; the remaining theorems (B2) and (B6) can easily be generalized. The details of the proof of Theorem 1 are then essentially the same as in [7]. This completes the outline of the proof of Theorem 1.

To the related question of the "smallest" prime *numbers* in a *residue* class mod $\mathfrak{f}$ we hope to come back soon.

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Purdue University