ing the temptation of stifling the reader under an undue burden of authoritarianism and slickness.

The authors in their preface to this volume intimate that this is not intended as an exhaustive treatment of the subject and this is indeed the case. Under these circumstances there is bound to be some controversy concerning the particular selection of topics made. Since the authors were clearly motivated in their choice of topics by geometric and not purely algebraic considerations, it might have been better if they had not included some of the purely algebraic sections (such as the section on chains of syzygies which is extremely sketchy and never used in an intrinsic way except in Appendix 7) and had used the space instead for giving geometric motivation and interpretation for more of the notions now presented in a purely formal setting, such as regular local rings, multiplicity theory, etc. Also some of the geometric interpretations given are so sketchy as to be almost meaningless, except to those already familiar with the subject from a geometric point of view. Since the authors are so admirably equipped to initiate the untutored into the connections between commutative algebra and geometry, it is a pity that they did not do more along these lines in this volume.

From the point of view of abstract algebra, or at least from the point of view of homological algebra, there is a glaring omission in this book. Modules are either not mentioned at all or just as a technical device in some few instances. The authors readily acknowledge this lack in their preface and I suppose their decision to play down this aspect of the subject can be defended on two grounds: (a) There are only a limited number of things you can have in a book; (b) module theory is not as central as ideal theory to their view of algebraic geometry. However, in the light of recent developments in both algebraic geometry and abstract algebra, I think a case can be made for having a book available which would approach commutative algebra from both the ideal theoretic and module theoretic points of view. While this treatise by Zariski and Samuel may not be as universal as one would like, it none the less does meet the need for an up-to-date book on ideal theory admirably and therefore deserves a wide audience.

M. AUSLANDER

Continuous transformations in analysis. By T. Radó and P. V. Reichelderfer. Springer-Verlag, Berlin, 1955. 7+442 pp. DM 59.60.

Any undergraduate student of calculus is aware of the fact that, if called upon to integrate a function of x, he may replace x by a suitable function of another variable u, and, by making the obvious substitutions, arrive at a new integral which is much more convenient to compute than the original. It is a rare student who observes that the mnemonic device built into the classical notation of calculus (a device which makes the above result obvious from the notational point of view) is not in itself a proof, and that the theorem is by no means obvious. The student of advanced calculus may even have forgotten this little matter when confronted with the comparable situation in regard to multiple integrals. Whether he has forgotten it or not, he is bound to be impressed by the complexity of the situation, and probably confused in the change from considerations in one dimension to several.

It has been said that on occasion a mature research mathematician will devote a portion of his energy to a sophisticated consideration of matters which bothered him as an undergraduate. If this is the case in this instance, the mathematical fraternity can be thankful that Radó and Reichelderfer found the subject of transformation of multiple integrals confusing to them in their mathematical infancy, for this monograph serves to illuminate many a dark corner.

Those who are not interested in the subject of continuous transformations in analysis as such will nevertheless be impressed by the wealth of topology as well as analysis which is required for a consideration of the generalizations of concepts with which they have been very familiar at the level of calculus.

The subtitle of this monograph is "With an Introduction to Algebraic Topology"; indeed the book contains a survey of set theoretic topology as well. The development of cohomology theory is novel and thorough, particularly as it applies to subsets of R_n (=euclidean n-space). The results are not unknown to a practicing topologist, but it is convenient to have a collection of all the relevant theorems on the cohomology of subsets of R_n within the covers of a single volume. Indeed (apart from the vexing inconvenience of a "shift in dimension" to be commented on later) the topological portions of the book might well be incorporated into an introductory course in analytic and algebraic topology.

So much for general comments.

The monograph is written in six parts: the first two are devoted essentially to background information in topology, the third to a survey of the required analytic tools, and the last three parts to the theory of continuous transformations.

Part I starts with a review of set theoretic topology, records the pertinent facts on abelian groups, and introduces the Mayer complex; that is, a sequence C^p of abelian groups and homomorphisms δ^p: C^p

$\to C^{p+1}$ such that $\delta^{p+1}\delta^p C^p = 0$ for $-\infty < p < \infty$. The authors then speak of a formal (Mayer) complex associated with a set $X \neq \phi$. The groups $C^p = 0$ for $p < 0$ while the elements in C^p $(p \geqq 0)$ are integral valued functions of $(p+1)$ variables in X. Their object is to proceed as far as is fruitful before imposing any topology on the set X. This is followed by a development of (reduced) cohomology theory akin to the approach adopted by Wallace and Spanier, and sketched by Radó in a note which appeared in the Proceedings of the American Mathematical Society [4 (1954), 244–246]. The standard cohomology group of dimension p is isomorphic to that of dimension $(p+1)$ in this book. Thus someone familiar with the standard theory is occasionally startled to see a manifestly false theorem unless his reflexes are conditioned to this "dimension shift." There are technical advantages to their approach in that there are no "identifications" prior to the essential step of taking the quotient group of cocycles modulo coboundaries; however, the reviewer is of the opinion that they should have taken the obvious steps to eliminate the dimension shift.

The application of the theory to compact subsets of R_n is made with great care, and includes such classical results as the Jordan theorem and the Brouwer invariance theorem.

Part II is concerned with the topological study of bounded continuous transformations $T: \boldsymbol{D} \to R^n$ where $\boldsymbol{D}$ is a bounded domain (=bounded connected open set). It is at this point that the authors introduce several "multiplicity functions" and show that the all important property of lower semi-continuity holds for them. These functions are generalizations of the naive multiplicity function $N(x, D) = \text{cardinal } [D \cap T^{-1}x]$ which is not lower semi-continuous in x for fixed $D \subset \boldsymbol{D}$.

The ideas are roughly as follows. Select an orientation for the unit sphere in R^n. For $x \in R^n$ and $\epsilon > 0$, let S_ϵ be the open ϵ-sphere with center x. For any domain D in $\boldsymbol{D}$ let $\mathfrak{G}$ be the collection of components of $D \cap T^{-1}S_\epsilon$ whose closures are in D. For $G \in \mathfrak{G}$ one has $(T|\overline{G}): (\overline{G}, \operatorname{Fr} G) \to (\overline{S}_\epsilon, \operatorname{Fr} S_\epsilon)$ and (in terms of the prior selected orientation, and the usual cohomological considerations) this map has a degree $\mu(G, \epsilon, x, D)$. Let $\mathfrak{G}^+(\mathfrak{G}^-)$ be the subcollection of $\mathfrak{G}$ for which $\mu > 0 (<0)$. Using the convention that an empty summation is zero, define the multiplicity functions K^+, K^- and K as follows:

$$K^+(x, D) = \lim_{\epsilon \to 0} \left[\sum \mu(G, \epsilon, x, D), G \in \mathfrak{G}^+ \right],$$

$$K^-(x, D) = \lim_{\epsilon \to 0} \left[- \sum \mu(G, \epsilon, x, D), G \in \mathfrak{G}^- \right]$$

and $K(x, D)=K^+(x, D)+K^-(x, D)$. If $K(x, D)<\infty$, let $\mu_\epsilon(x, D)=K^+(x, D)-K^-(x, D)$. These multiplicity functions play an essential role in the theory as will be seen in the discussion of later parts of the book.

Part III provides background information in real variables and goes on to consider functions of intervals in R^n. Since the book was written for analysts, when theorems are well known, but are to be used in the sequel, they are recorded as lemmas without proof. There are 44 such lemmas including such items as the Vitali covering theorem, the theorem of Lusin, and the lemma of Fatou. A function ϕ of intervals in R^n has a *derivative* $D(u, \phi)$ at $u\in R^n$ if $\lim \phi(Q)/LQ=D(u, \phi)$ for $u\in Q$ and $LQ\rightarrow 0$. The objects Q are n-cubes with edges parallel to the axes in R^n, and L stands for Lebesgue measure in R^n. The function ϕ is said to be *subadditive* in an open set G, if for any open interval I in G and any finite collection $\{I_k\}$ of open subintervals of I which are pair-wise disjoint it is true that $\sum\phi(I_k)\leqq\phi(I)$. It is shown that if ϕ is nonnegative, then subadditivity in G is a sufficient condition for the existence of the derivative a.e. in G. This result is of crucial importance in the theory.

Part IV, the longest section in the book, devotes its attention to general notions of bounded variation and absolute continuity. Given $T: \boldsymbol{D}\rightarrow R^n$ as before, an abstract multiplicity function $M(x, D)$, defined for $x\in R^n$ and each domain D in $\boldsymbol{D}$, is a nonnegative integral valued (∞ permissible) function satisfying four additional properties which, in this review will be termed axioms. The axioms are abstracted from certain conditions satisfied by the naive multiplicity function $N(x, D)$ of Part II; for example, $x\notin TD$ implies $M(x, D)=0$.

T is BVM(= of bounded variation with respect to M) in $\boldsymbol{D}$ if $M(x, \boldsymbol{D})$ is L summable. Thus there is a function $\mathfrak{M}(D)=\int M(x, D)dL$ defined over domains D in $\boldsymbol{D}$. It is shown that if T is BVM, then the derivative $D(u, \mathfrak{M})$ exists a.e. in $\boldsymbol{D}$, is L summable, and $\int_D D(u, \mathfrak{M})dL\leqq\mathfrak{M}(D)$ for $D\subset\boldsymbol{D}$.

T is ACM(=absolutely continuous with respect to M) if T is BVM and $\int_{\boldsymbol{D}} D(u, \mathfrak{M})dL=\mathfrak{M}(\boldsymbol{D})$.

The basic theorem is: If T is ACM in D, $H: R^n\rightarrow R^1$ is an L-measurable function, and $D\subset\boldsymbol{D}$, then $\int_D H(Tu)D(u, \mathfrak{M})dL=\int_{TD}H(x)M(x, D)dL$ if either of the integrals exists. This theorem is called herein, the *transfer formula*.

All this is pure analysis. The authors then show that the multiplicity functions K^+, K^- and K satisfy the axioms on M, and hence provide transfer formulas. They settle on the function K and define

T to be eBV (= essentially of bounded variation) if T is BVK, and eAC(= essentially absolutely continuous) if T is ACK.

It is shown that T is eBV if and only if it is BVK^+ and BVK^-; hence eBV implies the existence of the derivatives $D(u, \mathfrak{K})$, $D(u, \mathfrak{K}^+)$ and $D(u, \mathfrak{K}^-)$ a.e. in D. It is now possible to introduce the generalized Jacobian $J_e(u) = D(u, K^+) - D(u, K^-)$ for a.e. $u \in D$. It follows that $|J_e(u)| = D(u, \mathfrak{K})$. (Equalities, as usual, are to be read modulo sets of L-measure zero.)

If T is eAC in D and H: $R^n \to R'$ is L-measurable, then the general transfer formula provides that $\int_D H(Tu) | J_e(u) | dL = \int_{TD} H(x) K(x,D) dL$ if either of the integrals exists. Moreover, on considering the general transfer formula in respect to K^+ and K^- one obtains a like statement about the equality $\int_D H(Tu) J_e(u) dL = \int_{TD} H(x) \mu_e(x,D) dL$ if the integral on the left exists. (The naive multiplicity function N provides no analogue to the last statement roughly speaking because it does not take on negative values and hence is unsatisfactory from the point of view of generalizing the algebraic notion of Jacobian.) It is interesting to observe that if T is a homeomorphism then one gets the familiar transfer formula $\int_D H(Tu) | J_e(u) | dL = \int_{TD} H(x) dL$.

The authors go on to show that in other ways J_e behaves so as to deserve the name generalized Jacobian.

The chapter ends with a discussion of bounded variation and absolute continuity in the Banach sense, concepts which can be subsumed within the framework of the general theory of this part.

Part V begins with a proof of the fact that the various concepts of bounded variation, absolute continuity and generalized Jacobian, introduced in Part IV, are, for $n = 1$, respectively equivalent to the classical concepts found in real variables. Consequently, the notions introduced by the authors pass the acid test of being extensions of the usual concepts if $n = 1$. For $n = 2$, the only classic concept of pertinence available is the Jacobian, hence it would be gratifying to have an answer to the following question: If T is of class C' does the ordinary Jacobian J agree with the generalized Jacobian J_e? The reviewer could find no mention of this obvious question. However, using some of their theorems the reader can infer that the answer to this question is, as it should be, in the affirmative. (See pp. 374–377.) As a matter of fact, the matter may have been overlooked simply because it is the first question a layman in the field would ask, for they show that $J = J_e$ for Lipschitzian transformations and an even wider class which they introduce and call generalized Lipschitzian.

Part VI is concerned with the special features which help to make the theory much simpler in case $n=2$. A countability result, false for $n>2$, delayed the development of the general theory for many years. The authors go on to consider the concepts of bounded variation and absolute continuity introduced by Tonelli (the first attempt to generalize the classical notions of real variables) and investigate the interconnections with their own ideas. For the benefit of those interested in the field, an unproved conjecture is stated on page 433. From the point of view of applications, it is interesting to note that any Dirichlet transformation is eAC.

So much for a part by part summary. In general the book leaves something to be desired in a nonmathematical way.

The index is inadequate, and it is difficult to follow up the cross references, which are given in the form III.3.3 while the clues at the top of the page cut matters short with III.3. The printing does not live up to the high standards one has been led to expect of Springer-Verlag, and there are numerous broken characters and smudges (see, for example p. 212). The theorems are often easy to miss because their statements are not italicized. Finally the cohomological diagrams should have been left to the mercy of a printer (even a careless printer). Though they were apparently drafted with care, the results are sometimes shocking. All in all, though, these are probably petty grievances—the mathematical content certainly stands on its own feet, and is an outstanding source of information on an important and very difficult portion of modern mathematics.

J. W. T. YOUNGS

Leçons d'algèbre moderne. By P. Dubreil and M. L. Dubreil-Jacotin. Dunod, Paris, 1961. 11+393 pp. 35 NF.

The authors present a course in abstract algebra (the term "modern" is open to question, since almost all the material was already in the first edition of van der Waerden) designed for a program roughly at the level of the master's degree in this country. Their treatment is straightforward, usually aiming for proofs which involve the least complicated apparatus. This approach makes it possible to include a large number of results on assorted topics in fewer than four hundred pages.

Several notions which play an important part in modern mathematics, among them those of module and tensor product, are either given only passing notice or left out completely. These omissions may not be of great importance in the case of students completing their training with this course, but they are serious deficiencies for students