tions the reader does not get full information. For instance, the only book in the list on Legendre functions is that by Hobson (1931); the more recent books by MacRobert (1947), Lense (1950), Robin (1957–1959) are not brought to the reader's attention.

The general impression one gets of this book is that of a thorough and very lucidly written textbook which is well suited to systematic study from cover to cover and will reward the student not only with a knowledge of the functions presented in the book but also with an unusually clear presentation of the various methods used in acquiring this knowledge. All in all a valuable addition to the growing number of books devoted to special functions.

A. ERDÉLYI

Introduction to geometry. By H. S. M. Coxeter. John Wiley & Sons, New York, London, 1961. 15+443 pp. $9.95.

Geometry, in the Greek view synonymous with mathematics, was confronted with two competitors when mathematics revived in modern times, with algebra and analysis. But though more powerful the methods of algebra and analysis were still considered as philosophically inferior to that of geometry. And mathematical rigor remained associated with the names of Euclid and Archimedes. It is a fact that today geometry has lost much of its reputation. Geometry pursued along traditional lines is called old-fashioned, not only because of an apparent lack of rigor, but also on account of the alleged insignificance of its results. Coxeter presents classical style geometry in 22 chapters, which are reasonably self-contained, though tied together by a modern spirit of reinterpretation of classical matter. If geometry can be rewritten in a modern style without losing its classical character, is it fair to call it out of date? The answer of dogmatics to this rhetorical question will still be: yes, it is. They will emphasize this answer when they read the table of contents of the first chapter "Triangles": 1. Euclid, 2. Primitive concepts and axioms, 3. *Pons asinorum*, 4. The medians and the centroid, 5. The incircle and the circumcircle, 6. The Euler line and the orthocenter, 7. The nine-point circle, 8. Two extremum problems, 9. Morley's theorem. Of course they will never read this chapter (or the others either). If they are endowed with a sense of mathematical beauty, this is to be regretted. Fortunately there are people left, who like mathematical still-life. If they read this chapter they will admire not only the choice of subjects, but also the condensed style as opposed to the verbosity of many older geometry texts, and the compact lucid proofs in which every definition and conclusion is completely to the point. These are characteristics not only of the first chapter. They will make reading the book a pleasure to everybody who honestly tries to appreciate the subject itself in a positive sense. Another feature is the rich variety of subjects in the main text and in the exercises. Geometrical transformations and groups penetrate the interpretation as much as possible, axiomatics, and non-euclidean geometry are not neglected, there is even a short course of differential geometry included, and topology is represented by the four-color problem. The main feature, however, is didactical. Abstraction is approached not by blunt decree, but by a scale of seemingly tentative generalizations. Owing to this feature, Coxeter's book belongs to the very few from which textbook authors might learn how to write.

I mentioned that axiomatics has not been neglected. Axiomatic systems are admitted not for their own sake, but to serve special, well-defined aims, e.g. to prepare non-euclidean geometries. In the greater part of the book the approach is not from axiomatics. Many people will view this as a lack of rigor. They would have been satisfied if for conscience's sake the author had started with some axiomatic system, proving some theorems and quoting some others without proof ("which can easily be provided by the reader"), and finally reaching a stage where the reader voluntarily admits the usual basic facts of traditional geometry. I think mathematical rigor ought not to be interpreted in such a superficial and uniform way. Mathematical rigor knows different levels, and it is a matter of delicacy to know which level is adequate to which problem. Deriving Morley's theorem from axioms is as bad as proving some refined statements on order in the plane without axioms of order. People who believe there is a unique level of mathematical rigor usually forget that axiomatics is not the highest level as long as the language within the axioms is not formalized, or they reject any higher level than their own as crazy sophistication. The level on which geometry has been dealt with for centuries and which is still adequate to that kind of geometry, is that of local organization of the subject matter as opposed to the global organization viewed by axiomatics. Logical relations are tied between propositions, not to fasten them on a solid ground of primary propositions, but rather to approach a reasonable, but vague horizon of informal evidence. I agree with Coxeter's view on rigor in geometry and I believe that generally he has marvellously succeeded in finding the adequate context for the problems he has tackled.

Of course, there is still room for some criticism. With respect to the choice of subjects I restrict myself to mentioning one point only. Axiomatic and coordinate geometry coexist side by side in his book, often even in the same chapter, and conclusions are alternatively drawn, now in the one context and now in the other, though their isomorphism has not been proved or even stated as a problem. Seeing that this is one of the main points of Hilbert's *Grundlagen*, I would say that this is a serious lack of Coxeter's exposition.

Notwithstanding my all round positive appreciation of Coxeter's informal diction which mostly fits the contents very well, I cannot agree with all details. Too often he introduces a new notion N by some sentence S stating some property of N, but without telling whether this S means a definition of N or simply an intuitive statement S on some intuitively known thing N. (See among others the "definitions" of translations, p. 41, of glide reflections p. 43, of inversion p. 77.) Approaching a general notion by steps of successive generalization is a highly valuable didactical device, if finally a formal definition is formulated. But sometimes this last link is missing. See for instance p. 52 where a plane lattice is considered and some property of a certain parallelogram is noticed; then Coxeter continues: "For this reason the typical parallelogram is called a *fundamental region*. The shape of the fundamental region is far from unique. Any parallelogram will serve provided. . . . But there is no need for the fundamental region to be a parallelogram at all; for example we may replace each pair of opposite sides by a pair of congruent curves, as in Fig. 41d." This is a nice way of getting acquainted with fundamental regions, if it ends in a formal definition, which unfortunately is lacking. Too often if statements are made, it is not clear whether they are intuitive or whether the proof is left to the reader or delayed until later. Some vague formulations can cause misconceptions; on p. 50 the reader might conclude that there are no other transformation groups of the plane than the 17 crystallographic groups. On pp. 252–255 conics are dealt with in projective geometry, but it is not clear how conics are defined and whether they are related to the earlier conics of naive Euclidean geometry (pp. 115–118). On pp. 238–240 I would have preferred a more exact formulation of Desargues' theorem and of the proof of the uniqueness of the fourth point in a harmonic quadruple, i.e. an explanation which kind of accidental incidences are allowed and which ones are forbidden. The elegance of Coxeter's treatment of projective geometry is an illusion bought by the omission (with reference to some other textbook) of a derivation of the fundamental theorem from Pappus' theorem.

These are rather superficial remarks which do not touch essential points. My most serious criticism is against the (traditional) geometrical proof of Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$, (real θ). It does not stand the criterion I dared to advance for rigor in non-axiomatic geometry. The logical path to the horizon of evidence is disappearing in unknown deserts. The inexperienced reader will not be able to ascertain whether the derivation of this important theorem is a sham-proof or whether there is some trustworthy element in it.

I repeat these are exceptional obscurities in this unusually lucid work. There are a very few points I did not understand. One of them is the theorem on the non-existence of regular star tessellations (p. 63), and the other is what the golden section and Fibonacci numbers have to do with phyllotaxis (p. 169). Probably I missed the point.

In the few examples I quoted from Coxeter's book I judged that the style was too "geometrical." On the contrary coordinate and especially differential geometry are, to my feeling, not geometrical enough. In differential geometry where Coxeter is rather conventional, I would have preferred more intuitive methods.

An important part is played in Coxeter's exposition by the history of geometry. His concise historical notices witness a standard of historical exactness and understanding which equals the high mathematical standard of the work. The book should be recommended as a pleasant reading to mathematicians with a solid geometrical background and a liking for geometry. It should also be recommended as a textbook to be used in the class room. I am, however, not sure whether the gaps in formality of definitions and statements are serious enough to make it less appropriate to independent study.

Finally a few minor remarks:

p. 79, 5 from below: III 35 should be III 36.

p. 183, the title of 12.4 is probably mistaken, it does not match the content.

The numerous references to other books are often strange. Such a reference may point to a definition or a proof which is not provided for in the context, or it may give a hint for general reading, or it may acknowledge some authorship. But often the meaning of the reference is utterly obscure. The references to a Russian book of Yaglom are particularly strange. It is quite improbable that they have to provide for missing material or general reading so the reader is led to suppose that the statement in the context is due to Yaglom. However, the enunciations of the context are either trivial or well-known facts. E.g. p. 78: Every line through O (the center of inversion) is invariant as a whole, but not point by point [Yaglom 2, 173]. p. 86: each member of either pencil (of orthogonal circles) is orthogonal to every member of the other [Yaglom 2, 215–220]. p. 175: The affine propositions in Euclid are those which are preserved by parallel projection from one plane to another [Yaglom 2, p. 17]. I did not check too many among these references, but I noticed a few others of this kind: p. 78. If $OP>k/2$ (O the center of the inversion, k the radius of the invariant circle) the inverse of P can easily be constructed by the use of compasses only, without a ruler [Forder 1, p. 222]. The proof is given. It is not mentioned that the theorem is true without the restriction on OP. p. 92: A sphere with center N and radius NS inverts the plane σ (tangent in S) into a sphere σ' on NS as diameter [Johnson 1, p. 108].

H. FREUDENTHAL

Neuere Methoden und Ergebnisse der Ergodentheorie. By Konrad Jacobs. Ergebnisse der Mathematik und ihrer Grenzgebiete, neue Folge, Heft 29. Springer Verlag, Berlin, 1960. 6+214 pp. DM 49.80.

This is a concise and elegant introduction to some of the new methods and results of ergodic theory. Its table of contents (translated and annotated) runs as follows.

Introduction. (Motivation, basic definitions, and a bird's eye view of the entire subject; written for the non-expert.) 1, *Functional-analytic ergodic theory.* (Mean ergodic theorem, first for unitary operators on Hilbert space, ultimately for semigroups on Banach spaces; emphasis on almost periodicity; norm convergence for martingales.) 2, *Markov processes.* (The work of Doeblin; heavy use of such modern methods as the Riesz convexity theorem and the Krein-Milman theorem.) 3, *The individual ergodic theorem.* (Birkhoff's theorem, the Dunford-Schwartz generalization; the Hurewicz theorem; the almost everywhere martingale theorem.) 4, *Global properties of flows.* (Recurrence, ergodicity and mixing; decomposition into ergodic parts; flows under a function; the problem of invariant measure, Ornstein's solution.) 5, *Topological flows.* (Considerations involving both measure and topology; typically, the work of Krylov and Bogoliubov.) 6, *Topological investigations in the space of measure-preserving transformations.* (The work of Halmos and Rohlin.) 7, *Non-stationary problems.* (Random ergodic theorem, non-stationary Markov processes.) 8, *Functional-analytic methods.* (Zorn's lemma, topological and metric spaces, topological vector spaces and Banach spaces, semigroups, Banach lattices, Hilbert spaces.) 9, *Measure and integral. Special vector spaces.* (Fields of sets, measures, measurable transformations, integration, L^p spaces, convergence theorems, conditional expectations, product spaces. Chapters 8 and 9 are called an appendix; their purpose is to fill gaps in the reader's prerequisites.) *Bibliography.*