## BOOK REVIEW

*Vorlesungen über theoretische Mechanik.* By D. Morgenstern and I. Szabó. Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 112. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1961. xi+374 pp. DM 69.00.

Appearance of any book on mechanics in a mathematical series is so unusual as to attract notice. The celebrated Yellow Series has provided four or five such exceptions, each of particular merit, so the volume presently under review is doubly conspicuous. While three earlier volumes (by Whittaker, Lichtenstein, and Hamel) have been treatises, this one chooses to follow its immediate predecessor (by Siegel) in reproducing the contents of university courses. In every other respect it is unlike Siegel's book. Making no attempt at Siegel's rigor, elegance, and conciseness, the authors give a loose outline of mechanics as a whole, provided with numerous illustrations rather close to the surface.

The reader inclined to dismiss the book because at first glance it seems standard and elementary would be mistaken, for it is neither. While retaining the aspect of the old-fashioned German course of general mechanics, it is in fact one of the first textbooks to show the influence of the revival and rebirth of classical mechanics in the past fifteen years. Since it is mainly a collection of examples, this review can do no more than remark and illustrate its tone and trends.

First, it reflects the broadening and deepening of the concepts and methods by the American school, but it does so by casual comments rather than by structure. While mechanics for the most part seems to mean continuum mechanics to the authors, they nevertheless begin with separate treatments of mass-points and rigid bodies, leaving the reader with the idea that there are three different systems rather than a single, unified mechanics. They know that the principle of moment of momentum cannot be "proved," and they develop correctly the symmetry of the stress tensor (a sure sign of very old or very new influence). Their basic laws are the principles of mass, linear momentum, and moment of momentum. Noll's principle of material indifference is given prominent reference but not stated. "So as henceforth to avoid the common consideration of 'small elements,' which are often so ingenious that only an expert can understand them," the authors base most of their general arguments on the transport theorem, of which they give an unusually awkward formal proof.

Even on easier topics the authors have troubles. There seems to be some fascination about the names "Euler" and "Lagrange" as applied to co-ordinates. On p. 69 the authors cannot resist telling us, as does every elementary textbook, that "$x$ and $t$ go back to Euler . . . , $y$ and $t$ to Lagrange." On p. 346 they tell us this is wrong, but still they get the matter quite confused. I cannot see what use such remarks are unless they are true. Since this historical point has been straightened out some years ago, it would seem that the authors might either have given us the true story or passed over the whole matter in silence as being of no great importance.

Especially in the first part, the reader must face a telegraphic oracle, whose pronouncements are particularly annoying when false. E.g., on pp. 90–91 we read that for linearized elasticity "it follows from the theory of elliptic differential equations that solutions [corresponding to] all boundary conditions occurring in practice exist and are unique when $(m-1)m>0$, while Ericksen has shown that even for the simplest boundary-value problem uniqueness fails if $(m-1)m\leqq 0$. In all materials that really occur, $m>2$." Here $m$ is the reciprocal of Poisson's ratio. It would take a page to set all this right. I remark only (1) Ericksen showed that uniqueness of the displacement boundary-value problem fails if $1\leqq m\leqq 2$; (2) uniqueness of the stress boundary-value problem fails if $-1\leqq m\leqq 1$; (3) the work of Browder and Morrey, cited here, while certainly applying to the displacement boundary-value problem, does not in any obvious way refer to other problems; (4) while what is meant by "occurring in practice" depends on who is practicing, the theorem to which the authors seem to refer certainly says nothing about the classical problem of bodies in contact, which arises in the practice of some; (5) while "materials that really occur" is a strange phrase to run upon in a mathematics book, $m=2$ for ideal incompressible materials, and the condition that the stored-energy function be a definite quadratic form, for unconstrained materials, is $m>2$ or $m<-1$; (6) this last condition is both necessary and sufficient that the mixed boundary-value problem have a unique solution, for general regions.

While the first half of the book shows the greater signs of haste and carelessness, it shows also the greater originality. The entire §9 contains material occurring for the first time in a treatise, namely, Morgenstern's important researches on the position of the approximate theories of beams, plates, and membranes in regard to the three-dimensional theory. There follows a simple if not entirely comforting statement and proof of St. Venant's principle. Throughout the first half of the book are many interesting sidesteps. E.g., on p. 95 we find Bazley's inequality with an elegant new proof.

The second half of the book contains material in a standard German course on mechanics for engineers. Everything is written out in Cartesian co-ordinates. While sufficient time is taken to make the subject comprehensible, it is hard to see that the various linearizations for low speeds, slender bodies, etc., really deserve a place in a general introduction to mechanics. If they are the engineering applications of yesterday, why should they be those of tomorrow, and why should aeronautics, rather than flame propagation or meteorology or oceanography, be selected for "actuality"? Also this part of the book fails to reflect modern knowledge of the topics it does treat. We encounter the principle of energy on p. 223, after a quite misleading attempt on p. 85, but even so it is too special to be recognized as a principle that holds, e.g., in thermoelasticity. The "plausibility" argument in favor of the Stokes relation on p. 220, while it was good enough for our grandparents, will convince nobody today. The authors could have improved their treatment of fluid mechanics time and again by using material in the article of Serrin in Flügge's *Handbuch der Physik*, Springer-Verlag, Vol. 8. Part 1, 1959. For example, on p. 269 they tell that "the basic equations of continuum mechanics are insufficient to describe phenomena within a shock front," but this does not stop them from presenting on pp. 307–309 the old Becker theory, based not only on the Navier-Stokes equations but on use of a special value of the Prandtl number, while the work of Gilbarg and Paolucci (1953) shows Becker's solution to be quite untypical of the general theory.

Surprisingly for a book in the Yellow Series, there are a good many misprints, and the bibliography is quite inaccurate. St. Venant's name is regularly misspelled; Milne-Thomson is made into two persons by the aid of two gratuitous initials; etc.

Perhaps if the authors had written a more modern book, it would have been less well adapted to use in German instruction in mechanics, which remains heavily influenced by the requirements of pre-war engineering. The authors are doing service by introducing, if sporadically as well as gently, some of the newer ideas in mechanics. A welcome and unusual feature of this book is a historical appendix, based in part on recent researches.

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