BOOK REVIEW

Banach spaces of analytic functions. By Kenneth Hoffman. Prentice-Hall Series in Modern Analysis, Prentice-Hall, New York, 1962. 13+217 pp.

This is an attractive book. To bring together the most important facts about Hardy's H^p spaces (Definition: $f\in H^p$ if $f\in L^p(0, 2\pi)$ and all Fourier coefficients of negative index vanish) was itself a bright idea, and the idea has been carried through with spirit. The book falls naturally into three parts. Chapters I–III contain classical preliminaries. Chapters IV-IX present all the important "finished" results of the modern theory. And the final Chapter X, more than twice as long as any other, is devoted to recent work on the Banach algebra H^∞.

Each chapter of the middle part (with the exception of Chapter IX, which investigates three disjoint aspects of the Banach space H^p) is a tightly connected exposition centered about a single theorem. Since taken together these constitute the big theorems of the subject, they are worth citing in detail.

In Chapter IV (theorem of Szegö-Kolmogoroff-Kreĭn): Let f' be the (a.e.) derivative of the increasing function f. Then

$$\exp \frac{1}{2}\pi \int_{-\pi}^{\pi} \log f' = \inf \int |1 - g|^2 df,$$

with the inf taken over all trigonometric polynomials $g = \sum_{k\geq 1} a_k e^{ik\theta}$.

In Chapter V (theorem of Riesz-Herglotz-Nevanlinna): Identify $f\in H^1$ with the obvious analytic F in the open disk. Suppose $F(0) > 0$. Then $F = BSA$ uniquely, where $B(z)$ is a "Blaschke product"

$$= \prod \left(\frac{\bar{\alpha}_n}{|\alpha_n|} \frac{\alpha_n - z}{1 - \bar{\alpha}_n z} \right)$$

with $\sum(1 - |\alpha_n|) < \infty$, and $S(z)$ is a "singular function"

$$= \exp \int \frac{e^{i\theta} + z}{e^{i\theta} - z} dH(\theta)$$

with H decreasing and $H' = 0$ a.e., and $A(z)$ is an "outer function"

$$= \exp \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z} \log |f(\theta)| d\theta.$$

In Chapter VI (theorem of Beurling-Rudin): Let $\mathfrak{A}$ be the algebra of analytic functions in the disk with continuous boundary values.

Let K be a closed set of measure zero on the circle. Let B be a Blaschke product whose α_n accumulate in K. Let S be a singular function whose $H' \equiv 0$ outside K. Let $\mathcal{J}$ be the set of all functions gBS, with $g \in \mathcal{A} \equiv 0$ on K. Then $\mathcal{J}$ is a closed ideal in $\mathcal{A}$, and conversely every closed ideal has this form.

Chapter VII presents a similar structure theorem (also due to Beurling) for closed subspaces of H^2 invariant under multiplication by $\mathcal{A}$. Chapter VIII states the relation between H^p spaces on the disk and H^p spaces on the half-plane.

As we have already suggested, Chapter X has a different flavor from the others. Its subject, H^∞, is still at a primitive stage of development, although many curious facts are now known about it, some discovered by the author. One tends to wish that he had delayed publication of this last chapter for another year. He would then have been able to incorporate his most recent work on logmodular algebras [Acta Math. **108** (1962), 271–317] as well as Carleson's solution of Kakutani's celebrated "corona problem" [Ann. of Math. (2) **76** (1962), 541–546].

From the standpoint of classical analysis some of the proofs presented by the author may seem unnatural, for instance the highbrow Helson-Lowdenslager proof of the F. and M. Riesz theorem (if dF has all its Fourier coefficients of negative index vanish, then $dF = F'd\theta$) rather than the traditional complex-variable proof that appears in Zygmund's *Trigonometrical series*. On the other hand, from the standpoint of abstract functional analysis, many of the theorems in the book might have been presented simply as theorems about Dirichlet algebras. The author has deliberately chosen a nice compromise between these extremes. He always states a theorem first on the disk, but proves it in a style that will generalize. He ignores theorems about Dirichlet algebras that become uninteresting when specialized to the disk, for instance the theorem of Wermer about analyticity of "parts" [Duke Math. J. **27** (1960), 373–381]. At the end of each chapter there is a brief history of its main topics, which (in spite of the disclaimer in the author's preface) adds a great deal to the value of the chapter. There are also exercises, some of which contain important facts.

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