
Write

\[ f_N(x) = \max_{\{x_i\}} \sum_{i=1}^{N} g_i(x_i) \]

where the maximum is taken over the region \( x_i \geq 0, \sum_{i=1}^{N} x_i = x, \) with \( x > 0. \) Under what conditions on the sequence \( \{g_i(x)\} \) can we assert that \( f_N(x) \sim N \phi(x) \) as \( N \to \infty? \)

Using the functional equation technique of dynamic programming, we see that

\[ f_N(x) = \max_{0 \leq y \leq x} [g_N(y) + f_{N-1}(x - y)]. \]

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Consider a system of \( N \) simultaneous differential equations of the form

\[ \frac{d}{dt} x_i = g_i(x_1, x_2, \ldots, x_N), \] \( x_i(0) = c_i, \)

where the \( g_i \) are polynomials in the components \( x_i \) or, more generally entire functions.

Under what conditions on the \( g_i \) do there exist functions \( h_i(y_1, y_2, \ldots, y_k), \) \( k < N, \) entire as functions of the \( y_i, \) with the property that the functions of \( t \) defined by

\[ f_i = h_i(y_1, y_2, \ldots, y_k), \]

\( i = 1, 2, \ldots, k, \)

satisfy a set of simultaneous differential equations of the form

\[ \frac{d}{dt} f_i = G_i(f_1, f_2, \ldots, f_k), \]

where the \( G_i \) are entire functions of their arguments? When these new variables exist, how does one determine them?

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