cases thereof, have not been included. One may also wonder if so many pages spent on the preliminaries of information theory are not a bit extravagant. The book is also expansive on the pedagogic side. There are many examples, computations, tables and interesting pictures. Some important results appear in successive stages with increasing generality and depth, and variations on a theme are often presented. All this should make the content of the book easier to digest and to appreciate. The current tendency to conciseness of exposition, even on the textbook level, is not followed here, to the benefit of the reader.

This book can well serve as a basic course in probability theory, before a serious involvement with measure theory as required in any worthwhile study of stochastic processes. It has been said that probability theory is just a chapter of measure theory. This statement is not so much false as it is fatuous, as it would be to say that number theory is just a chapter of algebra. However, for a student of mathematics who wants to learn probability, there is need for a book which is not trivial on the one hand, and does not resemble chapters on measure and integration on the other. Of course Feller's well known *Introduction to probability theory and its applications*, Volume 1, can fit the bill, except for those who are eager for a general probability space and keep wondering about his Volume 2. (It may be disclosed here that Volume 2 will be a surprise to them.) There are also some mature readers who have no use for coins and dice, or even genes and particles. This book by Rényi may be what they have been looking for.

KAI LAI CHUNG

Introduction to differentiable manifolds. By Serge Lang. Interscience, New York, 1962. 10+126 pp. $7.00.

From modest beginnings in the eighteenth century, differential calculus has had a continuous increase of power and scope, culminating recently with the global theory of differential manifolds and mappings. This theory, basic to modern differential topology and geometry as well as classical physics, emerges from two decades of semi-secret existence with the publication of this definitive *Introduction.*

In addition to organizing in a report to the public fragments collected since 1936 (from original articles by H. Whitney, mimeographed notes by S. S. Chern, and J. Milnor, books by C. Chevalley, and G. de Rham, and many others), this text extends the global calculus to the infinite-dimensional case, and constitutes a natural sequel to the *Foundations of modern analysis* by J. Dieudonné [Academic Press, New York, 1960] which makes a parallel extension of the local Calculus. [See L. Nachbin's excellent review, Bull. Amer. Math. Soc. **67** (1961), 246]. In fact, the *Foundations* and the *Introduction* (weighing respectively 28 and 12 ounces) give a complete account of the foundations of the two century edifice of differential calculus.

The champion brevity of the *Introduction* is due to three stubborn talents of its author. The first is the uncompromising taste shown in the selection of topics: all of the fundamental notions are present, including six difficult existence theorems, while no applications, examples, or specialized theorems are to be found. The second is his unerring instinct for elegence: the most concise proof, the proper generality, the most exact notations. The third is a heartless determination to avoid repetition: every genuinely repetitious proof is left to the reader.

Like Dieudonné's *Foundations*, the *Introduction* is suitable for use as a textbook for advanced undergraduates or beginning graduate students. Here however the extreme brevity is a mixed blessing, providing on one hand important lessons in mathematical elegance, on the other a heavy burden by its complete lack of examples, exercises, repetitions, and motivations.

The primary content of the book has four parts.

Chapter II (Manifolds): In which the differentiable category is established, some special types of morphisms are discussed, and an original existence theorem for partitions of unity is proven.

Chapter III (Vector Bundles): In which the category of vector bundles and bundle mappings is presented, and the tangent functor is defined.

Both of these chapters are distinguished by the fact that their objects may be locally like Banach spaces, and the definitions of their morphisms, exact sequences, and sub-objects are chosen coherently from several distinct possibilities which coincide in the finite-dimensional case.

Chapter IV (Vector Fields and Differential Equations): In which the existence and uniqueness of global solutions of ordinary differential equations on Banach manifolds are discussed (beware of an error in the proof), the exponential map of a spray is defined, and the existence and uniqueness of tubular neighborhoods established. The main novelty here is the use of sprays, for which the author acknowledges the collaboration of R. Palais.

Chapters V and VI (Differential Forms and The Theorem of Frobenius): In which the exterior differential algebra of forms on a Banach manifold is defined, and the Poincaré Lemma and Theorem of Frobenius are proven. Here the exposition differs necessarily from the standard treatment of the finite-dimensional case, and is very pretty. Except for the theory of the Lie derivative, the main features of the standard theory are all present. The integration of forms is excluded of course.

In addition to these chapters, the book contains secondary material on differential calculus, including an elegant proof of the inverse function theorem (Chapter I), on Riemannian metrics, including their relation to sprays (Chapter VII), on the spectral theorem for Hermitian operators (Appendix I gives a complete exposition of the subject in 7 pages), and on the classical language of the finite-dimensional calculus (Appendix II).

The extra care required by the infinite-dimensional extension is more than compensated by the clarification of the standard theory it provides, and the intrinsic geometric intuition it teaches. Thus the author's claim is justified, that the generalization is achieved "at no extra cost." Nevertheless, many a reader will feel frustrated by the lack of a significant example of a differentiable manifold which is not finite-dimensional. An example (credited to J. Eells, Jr.) is mentioned in the foreword, however, which is of central importance in current applications of differential calculus, and strongly reinforces the author's choices of definitions.

Finally, the author should be rebuked for allowing several careless errors to appear in the book. Although these are mathematically insignificant, they obscure the most important virtue of the book: the really subtle pitfalls have been expertly skirted. In spite of this minor negligence, the *Introduction* at once provides the expert with a fundamentally reliable handbook in an area of current research, and the novice with an elegant exposition of a basic category.

RALPH ABRAHAM

Differential geometry and symmetric spaces. By S. Helgason. Pure and Applied Mathematics Series, Vol. 12. Academic Press, New York, 1962. 14+486 pp. $12.50.

The mathematical community has long been in need of a book on symmetric spaces. S. Helgason has admirably satisfied this need with his book *Differential geometry and symmetric spaces*. It is a remarkably well written book that takes the "and" in its title seriously in both a material and spiritual sense. Indeed, about the first third of the book is devoted to a concise exposition of the differential geometry of abstract manifolds and Lie groups. But in addition to this obvious physical fact, the author has, whenever possible, chosen to emphasize the geometric point of view rather than the algebraic. The end result