Historical notes are found at the ends of some sections and chapters; there are helpful exercises at the end of each section, and the bibliography is fairly extensive.

It should be pointed out that the role of resolutions in this book is secondary. They are introduced after the definitions of Ext and of Tor as computational devices. In MacLane's approach they are not (and should not be) basic to the definitions.

There is a great temptation to ask that this book include many more applications in many more areas than it does. However, if we keep in mind the fact that the title of the book is *Homology*, and that the author assumes that we all know by now why we should study homology, we can appreciate the selectivity which the author has displayed. In the light of recent developments, Chapters IX and XII may well be a little out of date, but MacLane points out in his introduction that the subject is still in a state of flux, and it's anybody's guess as to what the final word may be (if any). In any event, this is a book which can be given to a student with the assurance that if he absorbs the material in it he will have learned a great deal. Moreover, the students who have been reading this book have found it to be extremely helpful and clear.

## DAVID A. BUCHSBAUM

Introduction to differentiable manifolds. By Louis Auslander and Robert E. MacKenzie. McGraw-Hill, New York, 1963. 9+129 pp. \$9.95.

There seem to be two main routes toward an understanding of contemporary differential geometry and its allied disciplines. One way proceeds in historical order through the classical material on curves and surfaces in three-space, then on to global problems and the full development of manifold theory. The second approach starts in higher dimensions, leading directly from Euclidean spaces to the fundamentals of manifold theory. Although several good books have appeared recently following the first approach (for example, those by Willmore and Guggenheimer), there is a vital need for material presenting the intuitive background for the second. This book is a very welcome contribution to this goal, presenting in clear readable form of the basic concepts of the geometry of manifolds.

Chapters 1 and 2 introduce differentiable manifolds and their tangent spaces, proceeding from Euclidean spaces to submanifolds of Euclidean spaces and then on to abstract manifolds. To give further motivation for introducing the abstract objects the next chapter introduces the non-singular projective algebraic varieties as mani-

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folds. Chapter 4 presents the tangent bundle and vector fields, and discusses the connection with ordinary differential equations. Chapter 5 deals with the relatively delicate points involved in the definition of submanifolds and goes on to mention Riemannian metrics, although the book as a whole touches on Riemannian geometry only incidentally.

This is the halfway point. In effect the authors have sketched part of the standard foundation material, emphasizing the mapping point of view and omitting any significant mention of differential forms, tensor fields or of the formal differential operations. The notation used is suitable for the immediate goals, but will be incomplete when the book is used in a year-long graduate course, since this material should be done in greater detail there.

The last half of the book presents a variety of topics. Chapter 6 wisely dips into greater depth to prove that manifolds can be imbedded into Euclidean spaces. The next two chapters deal brilliantly with the fundamental concepts of Lie group theory, and give a proof of the basic theorem relating Lie subalgebras and Lie subgroups. This material has a reputation for difficulty, but the authors show its genuine simplicity. Here again, in a thorough graduate course the instructor should cover the more technical material that is not mentioned in the book, such as the closed subgroup theorem. In addition, the proof of the basic theorem is incomplete because a technical point in the global form of the Frobenius complete integrability theorem is omitted.

The next chapter on fiber bundles maintains this high expository quality, presenting the basic constructions that are useful in geometric applications in a clear, clean form. The last chapter, titled "Multilinear algebra," presents some of the algebraic facts that are useful in the more elaborate parts of differential geometry, but such a sudden change of pace from the rest of the book is not well-motivated. (For example, a demonstration, as convincing as that in the preceding three chapters, of the essential simplicity of tensor analysis and the theory of affine connections would have been of great value.)

In summary, the book should appeal to a broad spectrum of readers, particularly those who in the past have been cut off from the rich inspiration of geometry by the excessively formal, algebraic emphasis that has been placed on the subject. It is not the ideal text for a standard second year graduate course, but at the minimum should be used there as outside reading and as a source of examples and exercises (which are, indeed, a very good feature of the book). It would also combine very well with the recent book *Differential forms*,

## BOOK REVIEWS

by H. Flanders, in a course at the first year graduate level that would be of great interest to mathematically inclined physicists and engineers.

Robert Hermann

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Homology theory. An introduction to algebraic topology. By S. Wylie and P. J. Hilton. Cambridge University Press, New York, 1960. xv+484 pp. \$14.50.

The authors state their purpose and the intended scope of this book in the introduction as follows:

"This book has been written with the intention of providing an introduction to algebraic topology as it is practised today. The reader is not supposed at the outset to possess any knowledge of algebraic topology; indeed, even the reader with no knowledge of analytic topology or abstract algebra is provided, in the Background to Part I, with a synopsis of the facts that are taken for granted in the text. The treatment throughout has been subject to the consideration that, if the book is to serve its purpose, it must provide an account of the basic notions of algebraic topology intelligible to the mathematician inexperienced in the techniques and problems described. However, though the treatment is elementary, we have been more ambitious in our choice of material than is customary in elementary textbooks. It appears to us that the literature is rich in advanced textbooks and adequate in elementary introductory textbooks, but that the two types of book are not very effectively linked. Again, the advanced textbooks themselves fall into two classes which may broadly be described as classical and modern and the rapid shifts of emphasis which the subject has experienced make it difficult always to recognized classical arguments in their modern dress. We have tried to provide the links which we believe the student might find difficulty in providing for himself from a study of the available literature.

"Thus, while our beginning is quite elementary, we have been able, by omitting certain topics, particularly those treated canonically in classical works, to reach in later chapters the parts of the subjects which lie in the immediate foreground of present-day research."

In the opinion of the reviewer, they have been fairly successful in achieving this purpose. There is no other text treating algebraic topology that starts at the beginning and leads up to the topics of greatest interest in current research. On the one hand, there do exist the more advanced texts and specialized treatises dealing with special topics such as homotopy groups, fibre spaces, sheaves, manifolds, homological algebra etc., and on the other hand, there are the more

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