INVARIANT DOMAINS FOR KLEINIAN GROUPS[1]

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Communicated by Lipman Bers, December 2, 1963

If the limit set, Σ, of a properly discontinuous group, Γ, of fractional linear transformations of the Riemann sphere, S, contains more than two points, call Γ Kleinian. Otherwise, call Γ elementary. Let $\{\Omega_i\}$ be an enumeration of the components of Ω, the set of discontinuity. If O is a domain in S, i.e., O is open and connected, let $\Gamma(O)$ be the subgroup of Γ of elements which map O onto itself. If $\Gamma(O)=\Gamma$, call O an *invariant domain*. If $\Gamma(\Omega_i)=\{id\}$, call Ω_i an *atom*.

THEOREM 1. *If Γ possesses three disjoint invariant domains then Γ is cyclic.*

THEOREM 2. *Suppose Γ possesses an invariant component Ω_0. If $0\neq i\neq j\neq 0$, then $\Gamma(\Omega_i)\cap\Gamma(\Omega_j)$ is a nonloxodromic and nonhyperbolic cyclic group. If Ω_0 is simply connected this latter group is nonelliptic.*

THEOREM 3. *If Γ is a Kleinian group with two disjoint invariant domains, then there exists a maximal pair of disjoint invariant domains[2] each of which is simply connected. All noninvariant components of Ω are atoms.*

The author is grateful to Leon Greenberg for pointing out how the next theorem follows from the methods used in proving the previous theorems and, essentially, from a deep theorem of Nielsen and Fenchel on Fuchsian groups.

THEOREM 4. *If O_1 and O_2 are a maximal pair of disjoint invariant domains for a Kleinian group, Γ, then O_1/Γ and O_2/Γ are homeomorphic surfaces.*

Examples are given where (a) Ω and Σ are both connected and (b) where Γ possesses two invariant components *and* atoms.

The proofs follow from remarks of which the following are typical. (1) A closed set, invariant under Γ, contains Σ. (2) The components of the complement of a closed connected set are simply connected. (3) If O is a simply connected domain invariant under a loxodromic transformation, T, then there is a Jordan arc in O invariant under T

[1] Research supported by the Office of Naval Research.

[2] *Added in proof.* O_1 and O_2 are a maximal pair of disjoint invariant domains if whenever O_1' and O_2' are a pair of disjoint invariant domains such that $O_i\subset O_i'$, then $O_i=O_i'$ for $i=1, 2$.

connecting the fixed points of T. (4) If O_1 and O_2 are disjoint simply connected domains invariant under a loxodromic T, the corresponding arcs, as in (3), divide S into two Jordan regions, one or the other of which must contain any domain disjoint from O_1 and O_2. (5) If O is a simply connected domain invariant under an elliptic T, then O must contain a fixed point of T.

The examples are elaborations of the ideas in L. R. Ford, *Automorphic functions*, 2nd ed., Chelsea, 1951, pp. 55–59.

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DIFFERENTIABLE NORMS IN BANACH SPACES[1]

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Communicated by W. Rudin, January 20, 1964

1. **Introduction.** In [4, p. 28] S. Lang has asked whether or not a separable Banach space has an admissible norm of class C^1. In this note we indicate a proof of the following theorem, which characterizes those Banach spaces for which such a norm exists.

THEOREM 1. *A separable Banach space has an admissible norm of class C^1 if and only if its dual is separable.*

It follows from this theorem that not even $C(I)$ possesses an admissible differentiable norm.

2. **Preliminaries.** Let X be a Banach space with norm α; we write $S_\alpha = \{x \mid \alpha(x) = 1\}$ and $B_\alpha = \{x \mid \alpha(x) \leqq 1\}$. A norm in X is admissible if it induces the same topology as does α. The dual space is written X^* and the norm dual to α is denoted by α^*. An $f \in X^*$ is called a support functional to B_α at $x \in S_\alpha$ if $\alpha^*(f) = f \cdot x$; if f has norm 1, it is called a normalized support functional and is written ν_x. A norm is smooth if there is a unique normalized support functional to B_α at each $x \in S_\alpha$. The norm α is differentiable at $x \neq 0$ if there is an $\alpha'(x) \in X^*$ such that

$$\lim_{y \to x; y \neq x} \frac{|\alpha(y) - \alpha(x) - \alpha'(x) \cdot (y - x)|}{\alpha(y - x)} = 0$$

[1] Research partially supported by NSF Grant G-24471.