RESEARCH PROBLEMS

1. Peter Flor: *Matrix theory.*

For any square matrix A, let $\operatorname{per}(A)$ denote the permanent of A and $s(A)$, the sum of the elements of A.

Prove or disprove the following statement: "If M is any $n \times n$ matrix of real nonnegative numbers, and if k is any integer, $1 \leqq k \leqq n$, then

$$\sum (\operatorname{per}(B) - \operatorname{per}(C))(s(B) - s(C)) \geqq 0,$$

where B and C range independently over the $k \times k$ submatrices of M."

For the case of M being *doubly-stochastic*, the statement reduces to a conjecture of Holens (see [1]) which in turn would imply the affirmative solution of van der Waerden's famous problem on permanents (see e.g. [2]).

References

1. F. Holens, *Two aspects of doubly stochastic matrices: permutation matrices and the minimum of the permanent function*, Canad. Math. Bull. **7** (1964), 507–510.
2. M. Marcus and M. Newman, *On the minimum of the permanent of a doubly stochastic matrix*, Duke Math. J. **26** (1959), 61–72.

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2. Robert Spira: *Riemann hypothesis in function fields.*

It is easy to verify, by a method similar to Spira [1], that the Riemann hypothesis for the ζ-function for function fields (Weil [2]) holds if and only if $|\zeta(1-s)| > |\zeta(s)|$. Try to prove this inequality directly.

References

1. R. Spira, *An inequality for the Riemann zeta function*, Duke Math. J. **32** (1965), 247–250.
2. A. Weil, *On the Riemann hypothesis in function fields*, Proc. Nat. Acad. Sci. U.S.A. **27** (1941), 345–347.

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