

A NOTE ON SUPPORT POINTS

BY N. T. PECK

Communicated by Ernest Michael, May 27, 1968

Bishop and Phelps proved in [1] that every proper closed convex subset of a Banach space has "many" support points. (A *support point* of a convex set C in a real topological linear space E is a point x of C for which there is a nonzero continuous linear functional f on E with $f(x) = \sup_{c \in C} f(c)$.) At the end of their paper they asked whether "Banach space" can be replaced by "complete locally convex space" in the statement of this result. In [3], Klee settled this negatively by exhibiting a proper closed convex set in R^{\aleph_0} with no support points. The set in Klee's example is unbounded, and at the end of [3] the question was raised whether a bounded closed convex subset of a complete locally convex space must have support points. (Note that a bounded closed convex subset of R^{\aleph_0} is weakly compact and hence has support points. Indeed, if $\{B_i\}_{i=1}^{\infty}$ is any sequence of reflexive Banach spaces, every bounded closed convex subset of the product space $\prod_{i=1}^{\infty} B_i$ is weakly compact and therefore has support points.)

We settle Klee's question negatively:

THEOREM. *Let $\{B_i\}_{i=1}^{\infty}$ be any sequence of nonreflexive Banach spaces. Then in the product space $\prod_{i=1}^{\infty} B_i$ there is a closed bounded convex set which has no support points.*

The construction of the example rests on the theorem of James [2], that on every nonreflexive Banach space B , there is a continuous linear functional which does not assume its supremum on the unit ball of B . The detailed proof and related results will appear elsewhere.

REFERENCES

1. Errett Bishop and R. R. Phelps, *The support functionals of a convex set*, Proc. Sympos. Pure Math., vol. 7, Amer. Math. Soc., Providence, R. I., 1963, pp. 27–35.
2. R. C. James, *Weakly compact sets*, Trans. Amer. Math. Soc. **13** (1964), 129–140.
3. V. L. Klee, *On a question of Bishop and Phelps*, Amer. J. Math. **85** (1963), 95–98.

THE UNIVERSITY OF WARWICK, COVENTRY, ENGLAND