Let $C$ be the space of real $2\pi$-periodic continuous functions normed with the supremum norm. Let $P_n$ denote the subspace of trigonometric polynomials of degree $\leq n$. It is known [1] that the Fourier projection $F$ of $C$ onto $P_n$ is minimal; i.e., if $A$ is a projection of $C$ onto $P_n$ then $\|F\| \leq \|A\|$. We prove that $F$ is the only minimal projection of $C$ onto $P_n$. The proof is constructed by verifying the assertions listed below. Details will appear elsewhere.

**Assertion.** If there exists a minimal projection different from $F$, then there exist minimal projections $L$ and $H$, different from $F$ such that $\frac{1}{2}L + \frac{1}{2}H = F$.

The proof of this assertion utilizes Berman's equation,

$$F = \frac{1}{2\pi} \int_{-\pi}^{\pi} T_\lambda A T_\lambda d\lambda,$$

which is valid for any projection $A$ of $C$ onto $P_n$. Here $T_\lambda$ denotes the shift operator $(T_\lambda f)(x) = f(x+\lambda)$.

**Assertion.** There is a function $K(x, t)$ of two variables such that

(i) $K(x, t)$ satisfies (i) for each fixed $x$,
(ii) $K(t, t) \in P_n$ for each fixed $t$, and
(iii) $(L\phi)(x) = \int f(t)K(x, t)dt$.

This is proved by extending $A$ to its second adjoint, and applying the Radon-Nikodym theorem to the functionals $\phi(f) = (A\star f)(x)$.

Let $D_n$ denote the Dirichlet kernel. The next assertion follows from an examination of the roots of $K$ where $K$ is considered as a function of $x$.

**Assertion.** There is a function $g \in L^1$ such that $0 \leq g \leq 2$, and $K(x, t) = g(t)D_n(x-t)$.

**Assertion.** (i) $(1-g) \perp P_n$ and (ii) $(1-g) * D_n = 0$ where $*$ denotes convolution.

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Part (i) is immediate from the fact that $L$ is a projection. The minimality of $L$ is needed to prove part (ii).

Let $d(n, k) = \int |D_n(t)| e^{ikt} dt$.

**Assertion.** $d(n, k) \neq 0$ for $|k| > 2n$.

This result, when combined with the preceding assertion, will prove the theorem. The remainder of this paper pertains to proving that $d(n, k) \neq 0$.

**Assertion.**

$$d(n, k) = \frac{1}{\pi} \sum_{j=-k-n}^{k+n} \frac{1}{j} \frac{\beta^j - 1}{\beta^j + 1}$$

where $\beta = e^{2\pi i/2n+1}$.

**Assertion.** If $d(n, k) = 0$ then

$$\sum_{j=-k-n}^{k+n} \frac{1}{j} \sum_{t=1}^{2n} (-\beta)^t = 0.$$

Thus if $d(n, k) = 0$ we have a polynomial of degree $2n$ with rational coefficients which has $\beta$ as a root. We next derive a relation which must be satisfied by the coefficients of such a polynomial. The final step is to show that in our case this relation is not even satisfied modulo a convenient prime. The existence of the convenient prime is a consequence of the following extension of the Sylvester-Schur theorem.

**Assertion.** If $n$ and $k$ are integers satisfying $6 \leq k \leq n/2$, then at least two integers between $n-k+1$ and $n$ possess prime factors exceeding $k$.

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