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ON GENERALIZED COMPLETE METRIC SPACES

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The following remarks are of interest in connection with the research announcement [1]:

LEMMA. *A generalized metric space is the disjoint union of metric spaces such that each metric space is infinitely distant from every other metric space.*

PROOF. Note that $d(x, y) < \infty$ is an equivalence relation, and the equivalence classes obtained are metric spaces. Also, if the generalized space is complete, so is each metric space. Q.E.D.

Let $M = \bigvee_{\alpha \in A} M_\alpha$ denote the above partitioning. The Banach contraction principle becomes

PROPOSITION 1. *Let T be a strict contraction of a generalized complete metric space $M = \bigvee_{\alpha \in A} M_\alpha$, $0 \leqq q < 1$, $d(x, y) < \infty \Rightarrow d(Tx, Ty) \leqq qd(x, y)$. For each $\alpha \in A$, $\exists \beta \in A$ such that $T(M_\alpha) \subseteq M_\beta$. There is a unique periodic point of order n in each M_α such that $T^n(M_\alpha) \subseteq M_\alpha$.*

PROOF. Let $x, y \in M_\alpha$, $Tx \in M_\beta$. Then $d(x, y) < \infty \Rightarrow d(Tx, Ty) < \infty \Rightarrow Ty \in M_\beta$. Since T^n is a strict contraction of the complete metric space M_α, it has a unique fixed point, which is a periodic point of order n for T. Q.E.D.

The local contraction principle becomes

PROPOSITION 2. *Let T be a local contraction $(d(x, y) \leqq C \Rightarrow d(Tx, Ty) \leqq qd(x, y))$ of a complete generalized metric space. For each $\alpha \in A$, $x, y \in M_\alpha$, define $x \sim y$ iff $\exists x_0, \cdots, x_n \in M_0$ such that $x = x_0$, $y = x_n$, $d(x_i, x_{i+1}) \leqq C$ for $0 \leqq i \leqq n-1$. Then $\sim$ is an equivalence relation on each M_α; call the equivalence classes thus obtained C-components. T maps each C-component of M_α into a C-component of some M_β. There is a unique periodic point of order n in each C-component N of M_α such that $T^n(N) \subseteq N$.*

PROOF. Clearly $x \sim y$ is an equivalence relation; and if $x, y \in M_\alpha$ and $x \sim y$, let $x = x_0, \cdots, x_n = y$ be the chain. Then $d(Tx_i, Tx_{i+1}) \leqq qd(x_i, x_{i+1}) \leqq C$, and so $Tx \sim Ty$ in some M_β. The remainder is Theorem 1.4 of Bonsall (or Edelstein) of *On some fixed point theorems of functional analysis*. Q.E.D.

Several of Edelstein's and Rakotch's results go over analogously.

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