S. Smale has conjectured, in an unpublished paper, that the Morse Theory on Hilbert manifolds due to Palais and Smale [1], [4] can be extended to Banach manifolds. Under a different definition of nondegeneracy of critical points we have been able to make this extension. The result also extends Morse theory on Hilbert manifolds to a wider class of functions. I wish to thank R. Palais for several helpful suggestions.

Let $f$ be a real-valued $C^1$ function on a $C^1$ Banach manifold $X$. A critical point $x$ of $f$ is said to be weakly nondegenerate if there exists a neighborhood $U$ of $x$ and a hyperbolic linear isomorphism $L_x: T_x(X) \to T_x(X)$ such that in the coordinate system of $U$, $df_{x+v}(L_xv) > 0$ for all $x+v$ in $U$, $v \neq 0$. Then $T_x(X)$ splits into the direct sum of two invariant subspaces $T_x(X) = T_x(X)_+ \oplus T_x(X)_-$ such that the spectrum of $L_x$ on $T_x(X)_+$ lies in the right half plane and the spectrum of $L_x$ on $T_x(X)_-$ lies in the left half plane. The index of $f$ at $x$ is defined to be $\dim T_x(X)_-$, and this term is well defined. A nondegenerate critical point of a function on a Hilbert manifold is weakly nondegenerate.

**Theorem 1.** Let $f$ be a $C^2$ function on a $C^2$ paracompact manifold $X$ without boundary modeled on a separable Banach space $B$. We assume that $B$ has $C^2$ partitions of unity and a metric which is $C^2$ away from 0. If, in addition,

(a) $f$ satisfies condition (C) of Palais and Smale with respect to a complete Finsler metric on $X$, and

(b) $q > q'$ are not critical values, and all the critical points in $f^{-1}((q, q'))$ are weakly nondegenerate of finite index,

then there exists a homeomorphism $\theta: f^{-1}[q, -\infty) \approx f^{-1}[q', -\infty) \cup h_i$ where a handle $h_i$ of index $q_i$ is added for each one of the finite number of critical points $x_i \in f^{-1}((q, q'))$ of index $q_i$.

**Remark.** In the case of an infinite index, a similar result holds, provided that

$$df_{x+v}(L_xv) > \alpha(\|v\|_B)$$

for $0 \neq v \in T_x(M)_- \cap U$

where $\alpha$ is a continuous function from $R^+ \to R^+$.  

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THEOREM 2. Let $\eta$ be a vector bundle over a finite dimensional manifold $N$. Let the integral $J: L^p_2(\eta) \to \mathbb{R}$ be given by

$$J(s) = \int_N (1 + |A(s)|^2)^{p/2} + B(s) \, du \quad (p \geq 2)$$

where $A$ is a nonlinear (over-determined) elliptic system of order and weight $k$, $pk > \dim N$, and $B$ is of order $k-1$ and weight $pk$. Then $J$ is $C^\infty$ ($C^\infty$ for $p$ even) on the Sobolev space $L^p_k(\eta)$, and if the critical point $v$ has the properties:

(a) $v \in C^{k+a}(\eta)$ for any $a > 0$,
(b) the bilinear form $d^2J_v(\cdot, \cdot)$ extends to a nondegenerate form on $H_k(\eta)$,

then $v$ is a weakly nondegenerate critical point of $J$ with finite index.

REFERENCES

5. ———, Morse theory on Finsler manifolds (unpublished article).

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1 See Chapter 16 of [3].