Let \( S \) be a scheme and let \( G \) be a group scheme over \( S \). If \( \alpha: G \times X \to X \) is an action of \( G \) on \( X \) over \( S \) (cf. [4]), we say that \((X, \alpha)\) or simply \( X \) is a \( G \)-scheme over \( S \). The 'fixed point functor' \( h^G_X \) of \( G \) in \( X \) is defined as follows. For each \( S \)-scheme \( Y \), let \( Y_\alpha \) denote the trivial \( G \)-scheme \((F, \rho_2)\). Then

\[
h^G_X(Y) = \text{(set of } G\text{-linear } S\text{-morphisms } \varphi: Y_\alpha \to X)\text{.}
\]

**Theorem 1.** If \( \mathcal{C} \) is the category of locally noetherian \( S \)-schemes and quasicompact \( S \)-morphisms, \( X \) is a \( G \)-scheme in \( \mathcal{C} \), and \( G \) is flat over \( S \), then \( h^G_X \) is represented by a closed subscheme \( X^G \) of \( X \).

In this vast generality it is not to be expected that much detailed information about \( X^G \) can be obtained. Nevertheless, one does have the following 'rigidity' result when \( G \) is an abelian scheme over \( S \) (cf. [4]).

**Theorem 2.** Let \( G \) be an abelian scheme over \( S \) and let \( X \) be a connected locally noetherian \( G \)-scheme over \( S \). Then either \( X^G \) is empty or \( X^G = X \).

It is conceivable that this property could be used as the starting point for the general theory of abelian schemes, e.g., commutativity and Chow's theorem (cf. [3]) are easy consequences of Theorem 2.

For a deeper study of fixed point schemes, we restrict ourselves to the category of algebraic schemes over a field \( k \), acted upon by algebraic groups (i.e., smooth group schemes of finite type) over \( k \). One result, which is related to a special case of a recent result of G. Horrocks [2], is

**Proposition 3.** Let \( G \) be a linear algebraic group over \( k \). The largest \( k \)-closed normal subgroup \( H \) of \( G \) such that, for all proper connected \( G \)-schemes \( X \) over \( k \), \( X^H \) is connected is the unipotent radical of \( G \).

For smooth schemes and 'very good groups' one has:

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Proposition 4. If a linearly reductive linear algebraic group $G$ acts on a smooth algebraic scheme $X$ over $k$, then $X^G$ is smooth over $k$.

It seems to be an open question whether $X^o$ is smooth in the case of a semisimple group $G$ acting on a smooth $X$ over a field $k$ of characteristic $p > 0$. This is false for finite groups $G$ such that $p$ divides the order of $G$.

References